COURSE MATERIAL

IV Year B. Tech I- Semester

MECHANICAL ENGINEERING

AY: 2022-23

OPERATIONS RESEARCH

R18A0325

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MECHANICAL ENGINEERING

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

DEPARTMENT OF MECHANICAL ENGINEERING

CONTENTS

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

VISION

- To establish a pedestal for the integral innovation, team spirit, originality and competence in the students, expose them to face the global challenges and become technology leaders of Indian vision of modern society.

MISSION

- To become a model institution in the fields of Engineering, Technology and Management.
- To impart holistic education to the students to render them as industry ready engineers.
- To ensure synchronization of MRCET ideologies with challenging demands of International Pioneering Organizations.

QUALITY POLICY

- To implement best practices in Teaching and Learning process for both UG and PG courses meticulously.
- To provide state of art infrastructure and expertise to impart quality education.
- To groom the students to become intellectually creative and professionally competitive.
- To channelize the activities and tune them in heights of commitment and sincerity, the requisites to claim the never - ending ladder of **SUCCESS** year after year.

For more information: www.mrcet.ac.in

VISION

To become an innovative knowledge center in mechanical engineering through state-of-the-art teaching-learning and research practices, promoting creative thinking professionals.

MISSION

The Department of Mechanical Engineering is dedicated for transforming the students into highly competent Mechanical engineers to meet the needs of the industry, in a changing and challenging technical environment, by strongly focusing in the fundamentals of engineering sciences for achieving excellent results in their professional pursuits.

Quality Policy

- ✓ To pursuit global Standards of excellence in all our endeavors namely teaching, research and continuing education and to remain accountable in our core and support functions, through processes of self-evaluation and continuous improvement.
- ✓ To create a midst of excellence for imparting state of art education, industry-oriented training research in the field of technical education.

PROGRAM OUTCOMES

Engineering Graduates will be able to:

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and teamwork**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM SPECIFIC OUTCOMES (PSOs)

PSO1 Ability to analyze, design and develop Mechanical systems to solve the Engineering problems by integrating thermal, design and manufacturing Domains.

PSO2 Ability to succeed in competitive examinations or to pursue higher studies or research.

PSO3 Ability to apply the learned Mechanical Engineering knowledge for the Development of society and self.

Program Educational Objectives (PEOs)

The Program Educational Objectives of the program offered by the department are broadly listed below:

PEO1: PREPARATION

To provide sound foundation in mathematical, scientific and engineering fundamentals necessary to analyze, formulate and solve engineering problems.

PEO2: CORE COMPETANCE

To provide thorough knowledge in Mechanical Engineering subjects including theoretical knowledge and practical training for preparing physical models pertaining to Thermodynamics, Hydraulics, Heat and Mass Transfer, Dynamics of Machinery, Jet Propulsion, Automobile Engineering, Element Analysis, Production Technology, Mechatronics etc.

PEO3: INVENTION, INNOVATION AND CREATIVITY

To make the students to design, experiment, analyze, interpret in the core field with the help of other inter disciplinary concepts wherever applicable.

PEO4: CAREER DEVELOPMENT

To inculcate the habit of lifelong learning for career development through successful completion of advanced degrees, professional development courses, industrial training etc.

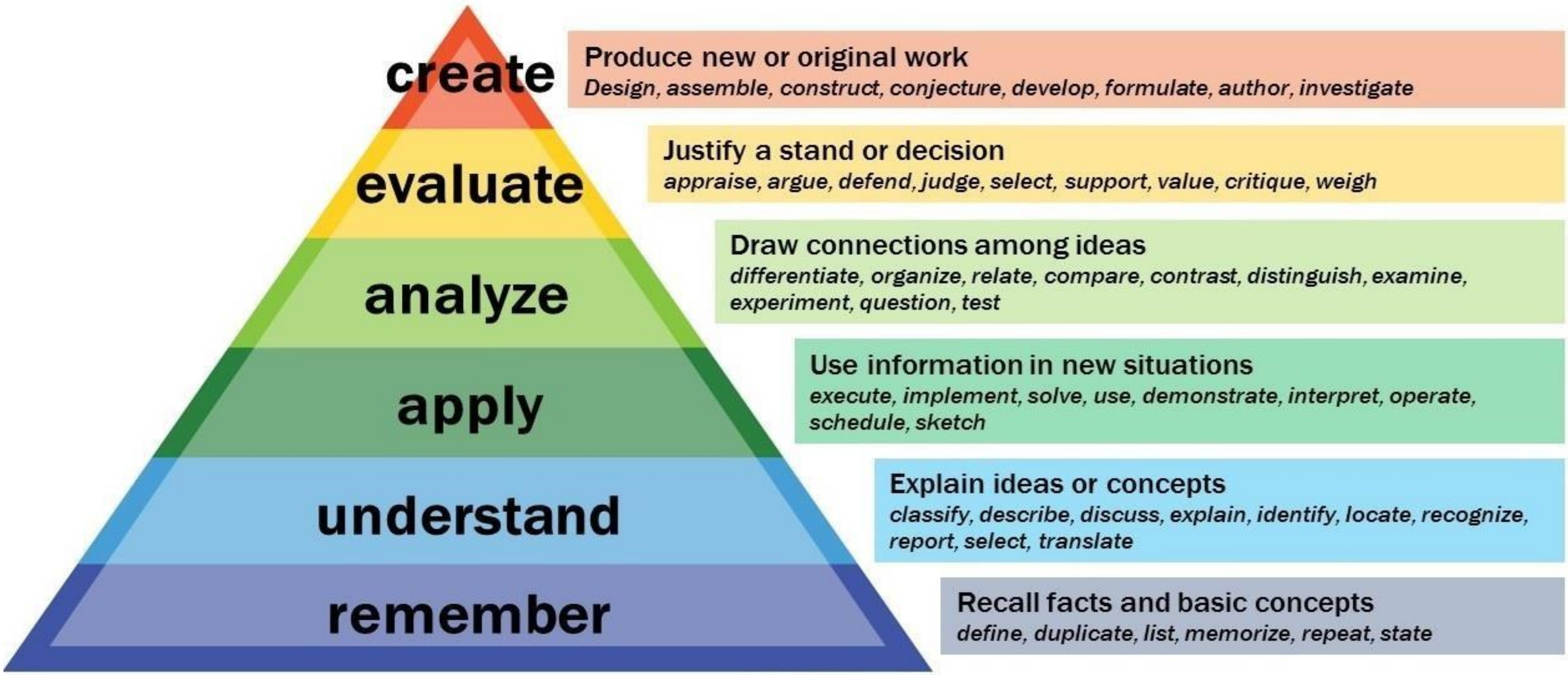
PEO5: PROFESSIONALISM

To impart technical knowledge, ethical values for professional development of the student to solve complex problems and to work in multi-disciplinary ambience, whose solutions lead to significant societal benefits.

Blooms Taxonomy

Bloom's Taxonomy is a classification of the different objectives and skills that educators set for their students (learning objectives). The terminology has been updated to include the following six levels of learning. These 6 levels can be used to structure the learning objectives, lessons, and assessments of a course.

1. **Remembering**: Retrieving, recognizing, and recalling relevant knowledge from long- term memory.
2. **Understanding**: Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.
3. **Applying**: Carrying out or using a procedure for executing or implementing.
4. **Analyzing**: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.
5. **Evaluating**: Making judgments based on criteria and standard through checking and critiquing.
6. **Creating**: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.



create
Produce new or original work
Design, assemble, construct, conjecture, develop, formulate, author, investigate
evaluate
Justify a stand or decision
appraise, argue, defend, judge, select, support, value, critique, weigh
analyze
Draw connections among ideas
differentiate, organize, relate, compare, contrast, distinguish, examine, experiment, question, test
apply
Use information in new situations
execute, implement, solve, use, demonstrate, interpret, operate, schedule, sketch
understand
Explain ideas or concepts
classify, describe, discuss, explain, identify, locate, recognize, report, select, translate
remember
Recall facts and basic concepts
define, duplicate, list, memorize, repeat, state

Course Syllabus

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

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IV Year B. Tech, ME-I SEM

(R18A0325) OPERATIONS RESEARCH
(PROFESSIONAL ELECTIVE 3)

Course Objectives:

1. Define and formulate linear programming problems and appreciate their limitations.
2. Solve linear programming problems using appropriate techniques and optimization solvers, interpret the results obtained and translate solutions into directives for action.
3. Conduct and interpret post-optimal and sensitivity analysis and explain the primal-dual relationship.
4. Develop mathematical skills to analyze and solve integer programming and network models arising from a wide range of applications.
5. Effectively communicate ideas, explain procedures and interpret results and solutions in simulation.

UNIT–I

Introduction: Development of OR – Definitions-Operation Research models– applications. **Resource Allocation:** Linear Programming Problem Formulation –Graphical solution – Simplex method –Artificial variables techniques -Big-M method

UNIT–II

Transportation Problem: Formulation – Optimal solution, unbalanced transportation problem –Degeneracy. **Assignment problem** –Formulation –Optimal solution - Variants of Assignment Problem-Traveling Salesman problem.

UNIT–III

Theory of Games: Introduction – Minimax (maximin) – Criterion and optimal strategy – Solution of games with saddle points – Rectangular games without saddle points – 2 X 2 games – dominance principle – m X 2 & 2 X n games -graphical method.

UNIT–IV

Replacement Analysis: Introduction – Replacement of items that deteriorate with time – when money value is not counted and counted – Replacement of items that fail completely, group replacement.**Inventory:** Introduction – Single item – Deterministic models – Purchase inventory models with one price break and multiple price breaks – shortages are not allowed

UNIT–V

Sequencing: Introduction – Flow **–**Shop sequencing – n jobs through two machines – n jobs through three machines – Job shop sequencing – two jobs through 'm' machines.

Simulation: Definition – Types of simulation models – phases of simulation– applications of simulation – Inventory and Queuing problems – Advantages and Disadvantages – Simulation Languages

TEXT BOOKS :

1 S.D.Sharma - Operations Research , Kedarnath, Ramnath 2015
2. Hiller &Libermann - Introduction to O.R , Mc Graw Hill 2011
3. Taha - Introduction to O.R , PHI 2010

REFERENCE BOOKS:

1. A.M.Natarajan,P.Balasubramani,A. Tamilarasi -Operations Research , Pearson . Education.
2. R.Pannerselvam - Operations Research ,PHI Publications 2006
3. J.K.Sharma- Operation Research , MacMilan 2010

Course Outcomes:

1. Student will be able to Identify and develop operational research models from the verbal description of the real system.
2. Understand the mathematical tools that are needed to solve optimization problems.
3. Develop a report that describes the model and the solving technique, analyses the results and propose recommendations in language understandable in Management Engineering.
4. Student able to understand Multi-criteria decision techniques, Decision making under uncertainty and risk, Game theory, and Dynamic programming.
5. Use mathematical software to solve the proposed simulation models.

Lecturer Notes

CONTENTS

COURSE COVERAGE SUMMARY

Units	Chapter No's In The Text Book Covered	Author	Text Book Title	Publishers	Edition
Unit-I Introduction to OR & Resource Allocation	1&2	R.Pannerselvam	Operations Research	Eastern Economy education	2
Unit-II Transportation Problem & Assignment problem	3 &4	R.Pannerselvam	Operations Research	Eastern Economy education	2
Unit-III Theory of Games	12	R.Pannerselvam	Operations Research	Eastern Economy education	2
Unit-IV Replacement Analysis & Inventory	7&13	R.Pannerselvam	Operations Research	Eastern Economy education	2
Unit-V Sequencing & Simulation	14	Kalavathy. S	Operations Research	Vikas publishing House Pvt.Ltd	4

UNIT 1

Introduction to OR

&

Resource Allocation

UNIT I

Operation Research is a relatively new discipline. The contents and the boundaries of the OR are not yet fixed. Therefore, to give a formal definition of the term Operations Research is a difficult task. The OR starts when mathematical and quantitative techniques are used to substantiate the decision being taken. The main activity of a manager is the decision making. In our daily life we make the decisions even without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. Operations Research tools are not from any one discipline. takes tools from different discipline such as Mathematics, Statistics, Economics, Psychology, Engineering etc, and Combines these tools to make a new set of knowledge for Decision Making.

DEFINITION of OR :

According to the Operational Research Society of Great Britain "Operational Research is the attack of modern science on complex problems arising in the direction and management of large systems of Men, Machines, Materials and Money in Industry, Business, Government and Defense. Its distinctive approach is to develop a Scientific model of the system, Incorporating measurements of factors such as Change and Risk, with which to predict and compare the outcomes of alternative Decisions, Strategies or Controls. The purpose is to help management determine its policy and actions scientifically".

According Morse and Kimball, "OR is A scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control"

.

According Miller and Starr, "O.R. is applied decision theory, which uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive, when he tries to achieve a thorough-going rationality in dealing with his decision problem".

Stages of Development of Operations Research: The stages of development of O.R. are also known as phases and process of O.R

Step I: Observe the problem environment

Step II: Analyze and define the problem

Step III: Develop a model

Step IV: Select appropriate data input

Step V: Provide a solution and test its reasonableness

Step VI: Implement the solution

Step I : Observe the problem Environment

Process Activities	**Process Output**
Site visits, Conferences, Observations, Research	Sufficient information and support to proceed

Step II : Analyze and define the problem

Process Activities	**Process Output**
Define: Use, Objectives, Limitations	Clear grasp of need for and nature of solution requested

Step III: Develop a mode

Process Activities	Process Output
Define inter relationships, Formulate equations, Use known O.R. Model, Search alternate Model	Models that works under stated environmental constraints

Step IV: Select appropriate data input

Process Activities	Process Output
Analyze: internal-external data, Facts Collect options Use computer data banks	Sufficient inputs to operate and test model

Step V: Provide a solution and test its reasonableness

Process Activities	Process Output
Test the model, find limitations, Update the model	Solution(s) that support current organizational goals

Step VI: Implement the solution

Process Activities	Process Output
Resolve behavioral issues, Sell the idea, Give explanations, Management involvement	improved working and Management support for longer run operation of model

Relationship between Manager/Decision Maker and O.R. Specialists

Steps in problem recognition, formulation and solution	Involvement O.R. specialist or manager
Recognize from organizational symptoms that a problem exists.	Manager
Decide what variables are involved; state the problem in quantitative relationships among the variables.	Manager and O.R. Specialist
Investigate methods for solving the problems as stated above; determine appropriate quantitative tools to be used	O.R. Specialist
Attempt solutions to the problems; find various solutions; state assumptions underlying these solutions; test alternative solutions.	O.R. Specialist
Determine which solution is most effective because of practical constraints within the organization; decide what the solution means for the organization.	Manager and O.R. Specialist
Choose the solution to be used.	Manager
Sell the decision to operating managers; get their understanding and cooperation.	Manager and O.R. Specialist

Tools and Techniques :

The common frequently used tools/techniques are mathematical procedures, cost analysis, electronic computation. However, operations researchers given special importance to the development and the use of techniques like

- Linear Programming
- Game Theory
- Decision Theory
- Queuing Theory
- Inventory Models and Simulation

In addition to the above techniques, some other common tools are

- Non-Linear Programming
- Integer Programming
- Dynamic Programming
- Sequencing Theory
- Markov Process,
- Network Scheduling (PERT/CPM),
- Symbolic Model,
- Information theory, and
- Value theory.

Linear Programming : This is constrained optimization technique, which optimize some criterion within some constraints. In Linear programming the objective function (profit, loss or return on investment) and constraints are linear. There are different methods available to solve linear programming

Game Theory : This is used for making decisions under conflicting situations where there are one or more players/opponents. In this the motive of the players are dichotomized. The success of one player tends to be at the cost of other players and hence they are in conflict.

Decision Theory : It is concerned with making decisions under conditions of complete certainty about the future outcomes and under conditions such that we can make some probability about what will happen in future.

Queuing Theory : This is used in situations where the queue is formed (for example customers waiting for service, aircrafts waiting for landing, jobs waiting for processing in the computer system, etc). The objective here is minimizing the cost of waiting without increasing the cost of servicing.

Inventory Model : It make a decisions that minimize total inventory cost. This model successfully reduces the total cost of purchasing, carrying, and out of stock inventory.

Simulation : It is a procedure that studies a problem by creating a model of the process involved in the problem and then through a series of organized trials and error solutions attempt to determine the best solution. Some times this is a difficult/time consuming procedure. Simulation is used when actual experimentation is not feasible or solution of model is not possible.

Non-linear Programming : This is used when the objective function and the constraints are not linear in nature. Linear relationships may be applied to approximate non-linear constraints but limited to some range, because approximation becomes poorer as the range is extended. Thus, the non-linear programming is used to determine the approximation in which a solution lies and then the solution is obtained using linear methods

Dynamic Programming : It is a method of analyzing multistage decision processes. In this each elementary decision depends on those preceding decisions and as well as external factors.

Information Theory : This analytical process is transferred from the electrical communication field to O.R. field. The objective of this theory is to evaluate the effectiveness of flow of information with a given system. This is used mainly in communication networks but also has indirect influence in simulating the examination of business organizational structure with a view of enhancing flow of information.

Applications of Operations Research

Accounting:

- Assigning audit teams effectively
- Credit policy analysis
- Cash flow planning
- Developing standard costs
- Establishing costs for byproducts
- Planning of delinquent account strategy

Construction:

- Project scheduling, monitoring and control
- Determination of proper work force
- Deployment of work force
- Allocation of resources to projects

Facilities Planning :

- Factory location and size decision
- Estimation of number of facilities required
- Hospital planning
- International logistic system design
- Transportation loading and unloading
- Warehouse location decision

Finance:

- Building cash management models
- Allocating capital among various alternatives
- Building financial planning models
- Investment analysis
- Portfolio analysis
- Dividend policy making

Manufacturing:

- Inventory control
- Marketing balance projection
- Production scheduling
- Production smoothing

Marketing:

- Advertising budget allocation
- Product introduction timing
- Selection of Product mix
- Deciding most effective packaging alternative

Organizational Behavior / Human Resources:

- Personnel planning
- Recruitment of employees
- Skill balancing
- Training program scheduling
- Designing organizational structure more effectively

Purchasing:

- Optimal buying
- Optimal reordering
- Materials transfer

Research and Development:

- R & D Projects control
- R & D Budget allocation
- Planning of Product introduction

Limitations of Operations Research

Distance between O.R. specialist and Manager : Operations Researchers job needs a mathematician or statistician, who might not be aware of the business problems. Similarly, a manager is unable to understand the complex nature of OR. Thus there is a big gap between the two personnel

Magnitude of Calculations : The aim of the O.R. is to find out optimal solution taking into consideration all the factors. In this modern world these factors are enormous and expressing them in quantitative model and establishing relationships among these require voluminous calculations, which can be handled only by machines

Money and Time Costs : The basic data are subjected to frequent changes, incorporating these changes into the operations research models is very expensive. However, a fairly good solution at present may be more desirable than a perfect operations research solution available in future or after some time

Non-quantifiable Factors : When all the factors related to a problem can be quantifiable only then OR provides solution otherwise not. The non-quantifiable factors are not incorporated in OR models. Importantly O.R. models do not take into account emotional factors or qualitative factors.

Implementation : Once the decision has been taken it should be implemented. The implementation of decisions is a delicate task. This task must take into account the complexities of human relations and behavior and in some times only the psychological factors.

Linear Programming:

- It is a special and versatile technique which can be applied to Advertising, Distribution, Refinery Operations, Investment, Transportation analysis and Production.

- It is useful not only in industry and business but also in non-profit sectors such as Education, Government, Hospital, and Libraries
- The linear programming method is applicable in problems characterizedby the presence of decision variables.
- The objective function and the constraints can be expressed as linear functions of the decision variables.
- The decision variables are in some sense, controllable inputs to the system being modeled.
- An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity, or minimizing cost or consumption
- There is always some practical limitation on the availability of resources like Man, Material, Machine, or Time for the system.
- These constraints are expressed as linear equations involving the decision variables.

L PP Formulation

- The L PP formulation is illustrated through a product mix problem. The product mix problem occurs in an industry where it is possible to manufacture a variety of products.
- A product has a certain margin of profit per unit, and uses a common pool of limited resources. In this case the linear programming technique identifies the products combination which will maximize the profit subject to the availability of limited resource constraints

Problem 1 : Suppose an Industry is manufacturing tow types of products P_1 and P_2.The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of P_1 and P_2. Formulate the problem in the form of linear programming model

Profit/Kg	**P_1 (Rs.30)**	**P_2 (Rs.40)**	**Total available Machine (hours/day)**
Machine 1	**3**	**2**	**600**
Machine 2	**3**	**5**	**800**
Machine 3	**5**	**6**	**1100**

Solution :

Constraints in this Problem are of “less than or equal to” type

Introduce the decision variable as follows

Let x_1 = amount of P_1

x_2 = amount of P_2

In order to maximize profits, we establish the objective function as

$$30X_1 + 40X_2$$

Since one Kg of P_1 requires 3 hours of processing time in machine 1 while the corresponding requirement of P_2 is 2 hours. So, the first constraint can be expressed as

$$3X_1 + 2X_2 \leq 600$$

Similarly, corresponding to machine 2 and 3 the constraints are

$$3X_1 + 5X_2 \leq 800$$

$$5X_1 + 6X_2 \leq 1100$$

In addition to the above there is no negative production, which may be represented algebraically as $X_1 \geq 0$; $X_2 \geq 0$

Thus, the product mix problem in the linear programming model is as follows:

Maximize : $30X_1 + 40X_2$

Subject to : $3X_1 + 2X_2 \leq 600$

$3X_1 + 5X_2 \leq 800$

$5X_1 + 6X_2 \leq 1100$

$X_1 \geq 0, \quad X_2 \geq 0$

Problem 2:

Formulation with Different Types of Constraints (L PP with different constraints)

A company owns two flour mills viz. A and B, which have different production capacities for high, medium and low quality flour. The company has entered a contract to supply flour to a firm every month with at least 8, 12 and 24 quintals of high, medium and low quality respectively. It costs the companyRs.2000 and Rs.1500 per day to run mill A and B respectively. On a day, Mill A produces 6, 2 and 4quintals of high, medium and low quality flour, Mill B produces 2, 4 and 12 quintals of high, medium and low quality flour respectively. How many days per month should each mill be operated in order to meet the contract order most economically.

Solution:

Let us define x_1 and x_2 are the mills A and B. Here the objective is to minimize the cost of the machine runs and to satisfy the contract order.

The linear programming problem is given by

Minimize

$$2000X_1 + 1500X_2$$

Subject to:

$$6X_1 + 2X_2 \geq 8$$

$$2X_1 + 4X_2 \geq 12$$

$$4X_1 + 12X_2 \geq 24$$

and $X_1 \geq 0$; $X_2 \geq 0$

Graphical Analysis of Linear Programming:

In this we use two models

1. Maximization Problems
2. Minimization problems

Maximization Problem :-

Maximize $Z = 30X_1 + 40X_2$

Subject to $3X_1 + 2X_2 \leq 600$

$3X_1 + 5X_2 \leq 800$

$5X_1 + 6X_2 \leq 1100$ and

$X_1 \geq 0, X_2 \geq 0$

Solution:-

$Z = 30X_1 + 40X_2$

Let us consider Equation (1) i.e

$3X_1 + 2X_2 \leq 600$

Put $X_1 = 0$, $X_2 = 0$, Equation (1)

We can get point = (0, 0)

Put $X_1 = 0$, in Equation (1) $X_2 = 300$

and $X_2 = 0$, in Equation (1) $X_1 = 200$

We can get points = (0, 300) and (200,0)

Let us consider Equation (2) i.e

$3X_1 + 5X_2 \leq 800$

Put $X_1 = 0$, in Equation (2) $X_2 = 160$ and $X_2 = 0$, in Equation (2) $X_1 = 266.66$

We can get points = (0, 160) and (266.66, 0)

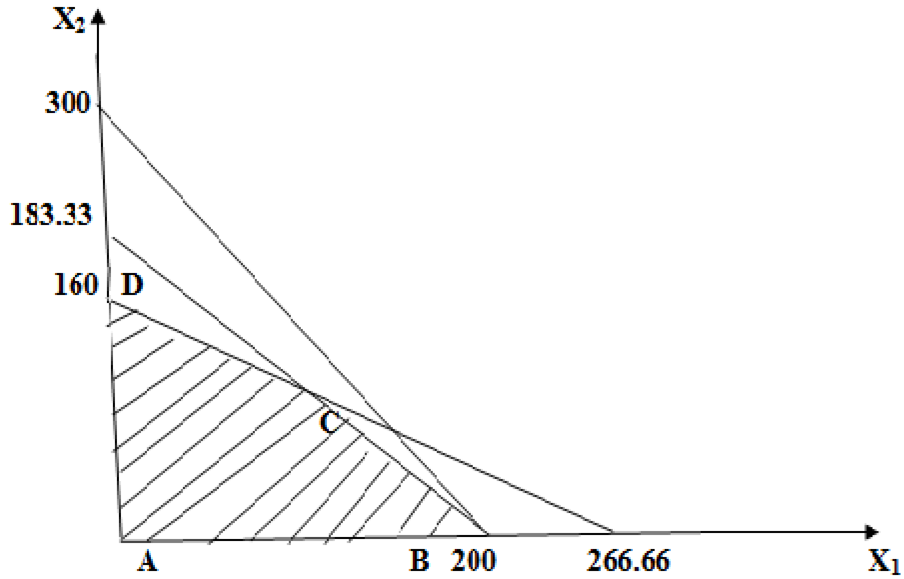
Let us consider Equation (3) i.e

$5X_1 + 6X_2 \leq 1100$

Put $X_1 = 0$, in Equation (3) $X_2 = 183.33$ and $X_2 = 0$, in Equation (3) $X_1 = 220$

We can get points = (0, 183.33) and (200, 0)

A, B, C, and D is feasible Region



A, B, C, and D is feasible Region

With help of graph from the Feasible Region we calculate Z_{MAX} Value

Key Terms :

Objective Function: Is a linear function of the decision variables representing the objective of the manager/decision maker.

Constraints: Are the linear equations or inequalities arising out of practical limitations.

Decision Variables: Are some physical quantities whose values indicate the solution.

Feasible Solution: Is a solution which satisfies all the constraints (including the non-negative) presents in the problem.

Feasible Region: Is the collection of feasible solutions.

Multiple Solutions: Are solutions each of which maximize or minimize the objective function.

Unbounded Solution: Is a solution whose objective function is infinite.

Infeasible Solution: Means no feasible solution.

SIMPLEX METHOD

The Linear Programming with two variables can be solved graphically. The graphical method of solving. Linear programming problem is of limited application in the business problems as the number of variables is substantially large.

If the linear programming problem has larger number of variables, the suitable method for solving is Simplex Method.

The simplex method is an iterative process, through which it reaches ultimately to the minimum or maximum value of the objective function.

The simplex method also helps the decision maker/manager to identify the following:

- Redundant Constraints
- Multiple Solutions
- Unbounded Solution
- Infeasible Problem

Basics of Simplex Method

The basic of simplex method is explained with the following linear programming problem.

Maximize: $60X_1 + 70X_2$

Subject to:

$2X_1 + X_2 \leq 300;$

$3X_1 + 4X_2 \leq 509;$

$4X_1 + 7X_2 \leq 812;$

$X_1, X_2 \geq 0$

Solution :

First we introduce the variables $S_3, S_4, S_5 \geq 0$ So that the constraints becomes equations, thus,

$$2X_1 + X_2 + 1S_3 + 0S_4 + 0S_5 = 300$$
$$3X_1 + 4X_2 + 0S_3 + 1S_4 + 0S_5 = 509$$
$$4X_1 + 7X_2 + 0S_3 + 0S_4 + 1S_5 = 812$$

Corresponding to the three constraints, the variables S_3, S_4, S_5 are called as slack variables.
Now, the system of equation has three equations and five variables..
There are two types of Solutions they are

- Basic Solutions
- Basic Feasible Solutions

Basic Solution

We may equate any two variables to zero in the above system of equations, and then the system will have three variables.
Thus, if this system of three equations with three variables is solvable such a solution is called as basic solution.
For example suppose we take $X_1 = 0$ and $X_2 = 0$, the solution of the system with remaining three variables is $S_3 = 300$, $S_4 = 509$ and $S_5 = 812$, this is a basic solution
The variables S_3, S_4, and S_5 are known as basic variables where as the variables x1, x2 are known as non-basic variables.
The number of basic solution of a linear programming problem is depends on the presence of the number of constraints and variables
For example if the number of constraints is “m” and the number of variables including the slack variables “n” there are at most basic solution.

$${}^{n}C_{n-m} = {}^{n}C_{m}$$

Basic Feasible Solution

A basic solution of a linear programming problem is called as basic feasible solutions if it is feasible it means all the variables are non-negative
The solution $S_3 = 300$, $S_4 = 509$ and $S_5 = 812$ is a basic feasible solution.
The number of basic feasible solution of a linear programming problem is depends on the presence of the number of constraints and variables.

For example if the number of constraints is m and the number of variables including the slack variables is n then there are at most basic feasible solutions

$$^{n}C_{n-m} = {}^{n}C_{m}$$

Every basic feasible solution is an extreme point of the convex set of feasible solutions and every extreme point is a basic feasible solution of the set of given constraints.

It is impossible to identify the extreme points geometrically if the problem has several variables but the extreme points can be identified using basic feasible solutions. Since one the basic feasible solution will maximize or minimize the objective function, the searching of extreme points can be carry out starting from one basic feasible solution to another.

The Simplex Method provides a systematic search so that the objective function increases in the cases of maximization progressively until the basic feasible solution has been identified where the objective function is maximized.

Problem :

Consider the following linear programming problem

Maximize : $60X_1 + 70X_2$

Subject to: $2X_1 + X_2 + 1S_3 + 0S_4 + 0S_5 = 300$;

$3X_1 + 4X_2 + 0S_3 + 1S_4 + 0S_5 = 509$;

$4X_1 + 7X_2 + 0S_3 + 0S_4 + 1S_5 = 812$

and $X_1, X_2, S_3, S_4, S_5 \geq 0$

Solution:-

In this problem the slack variables S_3, S_4, and S_5 provide a basic feasible solution from which the simplex computation starts. i.e S_3==300, S_4=509 and S_5=812. This result follows because of the special structure of the columns associated with the slacks.

If z represents profit, then z = 0 corresponding to this basic feasible solution.

We represent by C_B the coefficient of the basic variables in the objective function and by X_B the numerical values of the basic variable.

So that the numerical values of the basic variables are: X_{B1}=300, X_{B2}=509, X_{B3}=812.

The profit $Z = 60X_1+70X_2$ can also expressed as $z - 60X_1 - 70X_2 = 0$.

The simplex computation starts with the first compact standard simplex table as given below:

C_B	Basic Variables	C_j / XB	60 x_1	70 x_2	0 s_3	0 s_4	0 $s5$
0	s_3	300	2	1	1	0	0
0	s_4	509	3	4	0	1	0
0	s_5	812	4	7	0	0	1
	z		-60	-70	0	0	0

Table 1

In the objective function the coefficients of the variables are $C_{B1} = C_{B2} = C_{B3} = 0$. The topmost row of the Table 1 denotes the coefficient of the variables X_1, X_2, S_3, S_4, S_5 of the objective function respectively. The column under x_1 indicates the coefficient of x_1 in the three equations respectively. Similarly the remaining column also formed On seeing the equation $z = 60X_1+70X_2$ we may observe that if either X_1 or X_2, which is currently non-basic is included as a basic variable so that the profit will increase. Since the coefficient of X_2 is higher we choose X_2 to be included as a basic variable in the next iteration. An equivalent criterion of choosing a new basic variable can be obtained the last row of Table 1 i.e. corresponding to "Z" Since the entry corresponding to X_2 is smaller between the two negative values, X_2 will be included as a basic variable in the next iteration. However with three constraints there can be only three basic variables Thus, by bringing X_2 a basic variable one of the existing basic variables becomes non-basic. The question here is How to identify this variable?

The following statements give the solution to this question..
Consider the first equation i.e. $2X_1 + X_2 + 1S_3 + 0S_4 + 0S_5 = 300$

From this equation $2X_1+S_3 = 300 - X_2$

But $X_1 = 0$. Hence, in order that $S_3 \geq 0$ $300 - X_2 \geq 0$ i.e. $X_2 \leq 300$

Similarly consider the second equation i.e.

$$3X_1 + 4X_2 + S_4 = 509$$

From this equation $3X_1+S_4 = 509 – 4X_2$

But, $X_1=0$. Hence, in order that $S_4 \geq 0$ $509 - 4x2 \geq 0$ i.e. $X_2 \leq 509/9$

Similarly consider the third equation i.e.

$$4X_1 + 7X_2 + S_5 = 812$$

From this equation $4X_1+S_5 = 812-7X_2$

But $X_1 = 0$. Hence, in order that $S_5 \geq 0$

$$812 - 7X_2 \geq 0 \quad \text{i.e. } X_2 \leq 812 / 7$$

Therefore the three equation lead to

$$X_2 \leq 300, \quad X_2 \leq 509/9, \quad X_2 \leq 812/7$$

Thus $X_2 = \text{Min} (X_2 \leq 300, X_2 \leq 509/9, X_2 \leq 812/7)$ it means

$X_2 = \text{Min} (X_2 \leq 300/1, X_2 \leq 509/9, X_2 \leq 812/7) = 116$

Therefore $X_2 = 116$ If $X_2 = 116$, you may be note from the third equation

$$7X_2+S_5 = 812 \quad \text{i.e. } S_5 = 0$$

Thus, the variable S_5 becomes non-basic in the next iteration.

So that the revised values of the other two basic variables are

$$S_3=300-X_2 = 184 \qquad S_4 = 509 - 4 \times 116 =45$$

Refer to Table 1, we obtain the elements of the next Table i.e. Table 2 using the following rules:

1. We allocate the quantities which are negative in the Z- row. Suppose if all the quantities , the inclusion of any non-basic variable will not increase the value of the objective

function. Hence the present solution maximizes the objective function. If there are more than one negative values we choose the variable as a basic variable corresponding to which the Z value is least as this is likely to increase the more profit.

2. Let X_j be the incoming basic variable and the corresponding elements of the j^{th} row column be denoted by Y_1j, Y_2j and Y_3j respectively. If the present values of the basic variables are X_{B1}, X_{B2} and X_{B3} respectively, then we can compute.
   Min [X_{B1}/Y_{1j}, X_{B2}/Y_2j, X_{B3}/Y_{3j}] for $Y_{1j}, Y_{2j}, Y_{3j} > 0$.
   Note that if any $Y_{ij} \leq 0$, this need not be included in the comparison. If the minimum occurs Corresponding to X_{Br}/Y_{rj} then the r^{th} basic variable will become non-basic in the next iteration.

3. Using the following rules the Table 2 is computed from the Table 1.
   i.) The revised basic variables are S_3, S_4 and X_2. Accordingly, we make $C_{B1}= 0$, $C_{B2} = 0$ and $C_{B3} = 70$.
   ii.) As X_2 is the incoming basic variable we make the coefficient of x2 one by dividing Each element of row -3 by 7. Thus the numerical value of the element corresponding to X_1 is 4/7, corresponding to S_5 is 1/ 7 in Table 2.
   iii.) The incoming basic variable should appear only in the third row. So we multiply the third-row of Table 2 by 1 and subtract it from the first-row of Table 1 element by element. Thus the element corresponding to X2 in the first-row of Table 2 is 0. Therefore the element corresponding to X_1 is (2 - 1 x 4) / 7= 10 / 7 and the element corresponding to S_5 is (0 – 1 x 1 / 7) = -1/7 In this way we obtain the elements of the first and the second row in Table 2.

In Table 2 the numerical values can also be calculated in a similar way

C_B	Basic Variables	C_j X_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s_3	184	10/7	0	1	0	-1/7
0	s_4	45	5/7	0	0	1	-4/7
70	x_2	116	4/7	1	0	0	1/7
	zj-cj		-140/7	0	0	0	70/7

Table 2

Let C_{B1}, C_{B2}, C_{B3} be the coefficients of the basic variables in the objective function.
For example in Table 2 C_{B1}=0, C_{B2}=0 and C_{B3}=70. Suppose corresponding to a variable J, the quantity Zj is defined as $Z_j= C_{B1}, Y_1+ C_{B2}, Y_2j + C_{B3}Y_3j$.
Then the Z-row can also be represented as Zj - Cj.

For example: $Z_1 - C_1 = (10/7 \times 0) + (5/7 \times 0) + (70 \times 4/7)-60 = -140/ 7$
$Z_5 – C5 = (-1/7 \times 0) – (4/7 \times 0) +(1/7 \times 70) – 0 = 70/7$

1. Now we apply rule (1) to Table 2. Here the only negative Zj - Cj is Z_1-C_1 = -140/7

Hence X_1 should become a basic variable at the next iteration.

2. We compute the minimum of the ratio

$$\text{Min}\left(\frac{184}{\frac{10}{7}}, \frac{45}{\frac{5}{7}}, \frac{116}{\frac{4}{7}}\right) = \text{Min}\left(\frac{644}{5}, 63, 203\right) = 63$$

This minimum occurs corresponding to S_4, it becomes a non basic variable in next iteration

3. Like Table 2, the Table 3 is computed using the rules (i), (ii), (iii) as described above.

C_B	Basic Variables	C_j X_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s3	94	0	0	1	-2	1
60	x1	63	1	0	0	7/5	-4/5
70	x2	80	0	1	0	-4/5	3/5
	z_j-c_j		0	0	0	28	-6

Table 3

1. $Z_5 - C_5 < 0$ should be made a basic variable in the next iteration.
2. Now compute the minimum ratios

$$\text{Min}\left(\frac{94}{1}, \frac{80}{\frac{3}{5}}\right) = 94$$

Note: Since $Y_{25} = -4/5 < 0$, the corresponding ratio is not taken for comparison.
The variable s3 becomes non basic in the next iteration

3. From the Table 3, Table 4 is calculated following the usual steps.

C_B	Basic Variables	C_j X_B	60 x_1	70 x_2	0 s_3	0 s_4	0 s_5
0	s_5	94	0	0	1	-2	1
60	x_1	691/5	1	0	4/5	-1/5	0
70	x_2	118/5	0	1	-3/5	2/5	0
	z_j-c_j		0	0	6	16	0

Note that $z_j - c_j \geq 0$ for all j, so that the objective function can't be improved any further.
Thus, the objective function is maximized for x1 = 691/5 and x2=118/5 and
The maximum value of the objective function is 9944

Two Phase andBig M-Method

The simplex method was applied to linear programming problems with less than or equal to (≤) type constraints. Thus, there we could introduce slack variables which provide an initial basic feasible solution of the problem.
Generally, the linear programming problem can also be characterized by the presence of both less than or equal to " (≤) ' type or 'greater than or equal to " (≥)' type constraints.
In such case it is not always possible to obtain an initial basic feasible solution using slack variables.
The greater than or equal to type of linear programming problem can be solved by using the following methods:
1. Two Phase Method
2. Big M- Method

Problem:
Solve the following problem with Two phase method Minimize $12.5X_1 + 14.5X_2$ Subject to : $X_1 + X_2 \geq 2000$ $0.4X_1 + 0.75X_2 \geq 1000$ $0.075X_1 + 0.1X_2 \leq 200$ $X_1, X_2 \geq 0$

Solution:
Here the objective function is to be minimized; the values of X_1 and X_2 which minimized this objective function are also the values which maximize the revised objective function i.e. **Maximize : $-12.5 X_1 - 14.5X_2$**
We can multiply the second and the third constraints by 100 and 1000 respectively for the Convenience of calculation.
Thus, the revised linear programming problem is:

Maximize $-12.5X_1 - 14.5X_2$

Subject to: $X_1 + X_2 \geq 2000$

$40X_1 + 75X_2 \geq 100000$

$75X_1 + 100X_2 \leq 200000$ and $X_1, X_2 \geq 0$

Now we convert the two inequalities by introducing surplus variables S_3 and S_4 respectively. The third constraint is changed into an equation by introducing a slack variable S_5.
Thus, the linear programming problem becomes as

Maximize : $-12.5X_1 - 14.5X_2 = -25/2X_1 - 29/2\ X_2$

Subject to : $X_1 + X_2 - S_3 = 2000$

$40X_1 + 75X_2 - S_4 = 100000$

$75X_1 + 100X_2 + S_5 = 200000$ $X_1, X_2, S_3, S_4, S_5 \geq 0$

Even though the surplus variables can convert greater than or equal to type constraints into equations they are unable to provide initial basic variables to start the Simplex method calculation.
So we may have to introduce two more additional variables A_6 and A_7 called as artificial variable to facilitate the calculation of an initial basic feasible solution.

In this method the calculation is carried out in Two phases hence two phase method

Phase I :- In this phase we will consider the following linear programming problem

Maximize : $- A_6 - A_7$

Subject to: $X_1 + X_2 - S_3 + A_6 = 2000$

$40X_1 + 75X_2 - S_4 + A_7 = 100000$

$75X_1 + 100X_2 + S_5 = 200000$

and $X_1, X_2, S_3, S_4, S_5, A_6, A_7 \geq 0$

The initial basic feasible solution of the problem is $A_6 = 2000$, $A_7 = 100000$ and $S_5 = 200000$.

As the minimum value of the objective function of the Phase –I, is zero at the end of the Phase –I, calculation both A6 and A7 become zero.

C_B	Basic variables	C_j X_B	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 A_6	-1 A_7
-1	A_6	2000	1	1	-1	0	0	1	0
-1	A_7	100000	40	75	0	-1	0	0	1
0	S_5	200000	75	100	0	0	1	0	0
		z_j-c_j	-41	-76	1	1	0	0	0

Table 1

Here X_2 becomes a basic variable and A_7 becomes non basic variable in the next iteration. It is no longer considered for re-entry in the table.

C_B	Basic variables	C_j X_B	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5	-1 A_6
-1	A_6	2000/3	7/15	0	-1	1/75	0	1
0	X_2	4000/3	8/15	1	0	-1/75	0	0
0	S_5	200000/3	65/3	0	0	4/3	1	0
		z_j-c_j	-1/15	0	1	-1/75	0	0

TABLE 2

Then X_1 becomes a basic variable and A_6 becomes a non basic variable in the next iteration.

C_B	Basic variables	C_j X_B	0 x_1	0 x_2	0 s_3	0 s_4	0 s_5
0	x_1	10000/7	1	0	-15/7	1/35	0
0	x_2	4000/7	0	1	8/7	-1/35	0
0	s_5	250000/7	0	0	325/7	16/21	1
		z_j-c_j	0	0	0	0	0

Table 3

The calculation of Phase - I end at this stage. Note that, both the artificial variable have been removed and also found a basic feasible solution of the problem.

The basic feasible solution is :

$X_1 = 10000/7$

$X_2 = 4000/2$

$S_5 = 250000/7$

Phase II

The initial basic feasible solution obtained at the end of the Phase I calculation is used as the initial basic feasible solution of the problem.

In this Phase II calculation the original objective function is introduced and the usual simplex procedure is applied to solve the linear programming problem.

C_B	Basic variables	C_j X_B	-25/2 x_1	-29/2 x_2	0 s_3	0 s_4	0 s_5
-25/2	x_1	10000/7	1	0	-15/7	1/35	0
-29/2	x_2	4000/7	0	1	8/7	-1/35	0
0	s_5	250000/7	0	0	325/7	5/7	1
		$z_j - c_j$	0	0	143/14	2/35	0

Table 1

In this Table 1 all $Z_j - C_j \geq 0$ the current solution maximizes the revised objective function.

Thus, the solution of the problem is:

$X_1 = 10000/7 = 1428$

$X_2 = 4000/7 = 571.4$

and The Minimum Value of the objective function is: 26135.3

Big –M Method :

In this method also we need artificial variables for determining the initial basic feasible solution

Maximize: $-12.5X_1 - 14.5X_2$

Subject to:

$X_1 + X_2 - S_3 = 2000$

$40X_1 + 75X_2 - S_4 = 100000$

$75X_1 + 100X_2 + S_5 = 200000$

and $X_1, X_2, S_3, S_4, S_5 \geq 0$.

Introduce the artificial variables A6 and A7 in order to provide basic feasible solution in the second and third constraints.

The objective function is revised using a large positive number say M

Thus, instead of the original problem, consider the following problem i.e

Maximize : $-12.5 X_1 - 14.5X_2 - M (A_6 + A_7)$

Subject to: $X_1 + X_2 - S_3 + A_6 = 2000$

$40X_1 + 75X_2 - S_4 + A_7 = 100000$

$75X_1 + 100X_2 + S_5 = 200000$

and $X_1, X_2, S_3, S_4, S_5, A_6, A_7 \geq 0$

The coefficient of A_6 and A_7 are large negative number in the objective function.
Since the objective function is to be maximized in the optimum solution, the artificial variables will be zero.

Therefore, the basic variable of the optimum solution are variable other than the artificial variables and hence is a basic feasible solution of the original problem.
The successive calculation of simplex tables is as follows

C_B	Basic variables	C_j X_B	-12.5 x_1	-14.5 x_2	0 s_3	0 s_4	0 s_5	-M A_6	-M A_7
-M	A_6	2000	1	1	-1	0	0	1	0
-M	A_7	100000	40	75	0	-1	0	0	1
0	S_5	200000	75	100	0	0	1	0	0
		z_j-c_j	-41M +12.5	-76M +14.5	M	M	0	0	0

Table 1

Since M is a large positive number, the coefficient of M in the Zj – Cj row would decide the entering basic variable. As -76M < - 41M, X_2 becomes a basic variable in the next iteration replacing A_7.The artificial variable A_7 can't be re-entering as basic variable.

C_B	Basic variables	C_j X_B	-12.5 x_1	-14.5 x_2	0 s_3	0 s_4	0 s_5	-M A_6
-M	A_6	2000/3	7/15	0	-1	1/75	0	1
-14.5	X_2	4000/3	8/15	1	0	-1/75	0	0
0	S_5	200000/3	65/3	0	0	4/3	1	0
		z_j-c_j	-7/15M +143/30	0	M	-M/75 +29/150	0	0

Table 2

Now X_1becomes a basic variable replacing A_6, Like A_7 the variable A_6 also artificial variable so it can't be re-entering in the table.

C_B	Basic variables	C_j X_B	-12.5 x_1	-14.5 x_2	0 s_3	0 s_4	0 s_5
-12.5	x_1	10000/7	1	0	-15/7	1/35	0
-14.5	x_2	4000/7	0	1	8/7	-1/35	0
0	s_5	250000/7	0	0	325/7	16/21	1
		z_j-c_j	0	0	143/14	2/35	0

Table 3

Hence The optimum solution of the problem is

$X_1 = 10000/7$

$X_2 = 4000/7$ and

The Minimum Value of the Objective Function is: 26135.3

Tutorial Questions

1. Let us consider a company making single product. The estimated demand for the product for the next four months are 1000,800,1200,900 respectively. The company has a regular time capacity of 800 per month and an overtime capacity of 200 per month. The cost of regular time production is Rs.20 per unit and the cost of overtime production is Rs.25 per unit. The company can carry inventory to the next month and the holding cost is Rs.3/unit/month the demand has to be met every month. Formulate a linear programming problem for the above situation.

2. What are the advantages and applications of OR

3. Solve the following LPP by Big-M penalty method Minimize $Z = 5X_1 + 3X_2$
S.T $2X_1 + 4X_2 = 12$, $2X_1 + 2X_2 = 10$, $5X_1 + 2X_2 = 10$ and $X_1, X_2 \leq 0$

4. Solve the following LP problem using graphical method

Maximize $Z == -X_1 + 2X_2$

Subjected to $X_1 - X_2 \leq -1$

$-0.5X_1 - X_2 \leq 2$ $x_1, x_2 \geq 0$

5. Explain what is meant by degeneracy in LPP? How can this be solved?

Assignment Questions

1. Solve the following LP problem by two phase method. Maximize $Z = 5x_1+3x_2$

   subjected to $3x_1+2x_2 \geq 3$

   $x_1+4x_2 \geq 4$

   $x_1+x_2 \leq 5$

   $x_1+x_2 \geq 0$

2. Solve the following LPP problem by Two phase method Max $Z=2x_1+3x_2+5x_3$

   Subjected to

   $3x_1+10x_2+5x_3 \leq 15$

   $33x_1- 10x_2+9x_3 \leq 33$

   $x_1+2x_2+3x_3 \geq 4$

   $x_1, x_2, x_3 \geq 0$

3.a) Define the LPP. Give an example

b) Solve the following LPP using graphical method and verify by Simplex method Maximize

$Z=10x_1+8x_2$

$x_2 \leq 500$ and

$x_1 \leq 300$ $\quad x_1, x_2, \geq 0$

4. A firm produces three types of biscuits A,B,C it packs them in arrangement of two sizes 1 and 11. The size 1 contains 20 biscuits of type A, 50 of type B and 10 of type C. the size 11 contains 10 biscuits of type A, 80 of type B and 60 of type C. A buyer intends to buy at least 120 biscuits of type A, 740 of type B and 240 of type C. Determine the least number of packets he should buy. Write the dual LP problem and interrupt your answer

5. Explain what is meant by degeneracy in LPP? How can this be solved?

6. Solve the following LP problem by graphically

   Maximize $Z=2x_1+x_2$

Subjected to $X_1+2X_2 \leq 10$,

$X_1+X_2 \leq 6$,

$X_1-X_2 \leq 2$,

$X_1 - 2X_2 \leq 1$ $\quad x_1,x_2 \geq 0$

UNIT 2

Transportation Problem

&

Assignment problem

UNIT II
Transportation Problems

A special class of linear programming problem is Transportation Problem, where the objective is to minimize the cost of distributing a product from a number of sources (e.g. factories) to a number of destinations (e.g. warehouses) while satisfying both the supply limits and the demand requirement. The transportation model can be extended to areas other than the direct transportation of a commodity, including among others, inventory control, employment scheduling, and personnel assignment.
The problem has more constraints and more variables. So, it is not possible to solve such a problem using simplex method. This is the reason for the need of special computational procedure to solve transportation problem.

Transportation Algorithm:- The steps of the transportation algorithm are exact parallels of the simplex algorithm, they are:
Step 1: Determine a starting basic feasible solution, using any one of the following three methods
1. North West Corner Method
2. Least Cost Method
3. Vogel Approximation Method

Step 2: Determine the optimal solution using the following method
1. MODI (Modified Distribution Method) or UV Method.

The special structure of the transportation problem allows securing a non artificial basic feasible solution using one the following three methods.
1. North West Corner Method (NWCM)
2. Least Cost Method (LCM)
3. Vogel Approximation Method(VAM)

The difference among these three methods is the quality of the initial basic feasible solution they produce, in the sense that a better that a better initial solution yields a smaller objective value.

Generally the Vogel Approximation Method produces the best initial basic feasible solution, and the North West Corner Method produces the worst, but the North West Corner Method involves least computations

North West Corner Method :

The method starts at the North West (upper left) corner cell of the tableau (variable X_{11}).

Step -1: Allocate as much as possible to the selected cell, and adjust the associated amounts of capacity(supply) and requirement (demand) by subtracting the allocated amount.

Step -2: Cross out the row (column) with zero supply or demand to indicate that no further assignments can be made in that row (column). If both the row and column becomes zero simultaneously, cross out one of them only, and leave a zero supply or demand in the uncrossed out row (column).

Step -3: If exactly one row (column) is left uncrossed out, then stop. Otherwise, move to the cell to the right if a column has just been crossed or the one below if a row has been crossed out. Go to step -1.

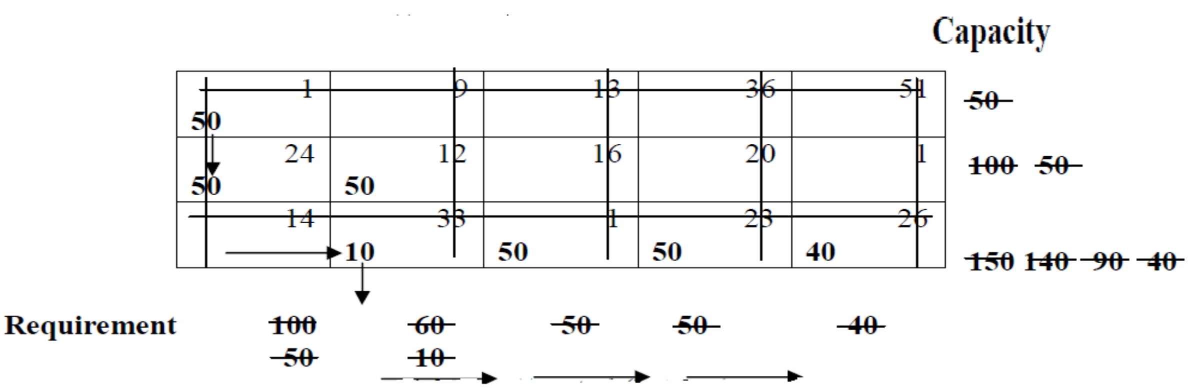
Problem1 :

Consider the problem discussed in Example 1.1 to illustrate the North West Corner Method of determining basic feasible solution

	Retail Agency					
Factories	1	2	3	4	5	Capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Solution :

The allocation is shown in the following tableau:



The arrows show the order in which the allocated (bolded) amounts are generated.

The starting basic solution is given as

$X_{11} = 50$, $X_{21} = 50$, $X_{22} = 50$ $X32 = 10$, $X_{33} = 50$, $X_{34} = 50$, $X_{35} = 40$

The corresponding transportation cost is

50 x 1 + 50 x24 + 50 x 12 + 10 x 33 + 50 x 1 + 50 x 23 + 40 x 26 = 4420

It is clear that as soon as a value of X_{ij} is determined, a row (column) is eliminated from.

Further Consideration. The last value of X_{ij} eliminates both a row and column. Hence a feasible solution computed by North West Corner Method can have at most m + n – 1 positive X_{ij} if the transportation problem has m sources and n destinations

Least Cost Method :

The LCM is also known as matrix minimum method in the sense we look for the row and the column corresponding to which C_{ij} is minimum.

This method finds a better initial basic feasible solution by concentrating on the cheapest routes.Instead of starting the allocation with the northwest cell as in the North West Corner Method, we start by allocating as much as possible to the cell with the smallest unit cost. If there are two or more minimum costs then we should select the row and the column. Corresponding to the lower numbered row.If they appear in the same row we should select the lower numbered column

We then cross out the satisfied row or column, and adjust the amounts of capacity and requirement accordingly.
If both a row and a column is satisfied simultaneously, only one is crossed out. Next, we look for the uncrossed-out cell with the smallest unit cost and repeat the process until we are left at the end with exactly one uncrossed-out row or column.

Problem2 :

Determine the initial basic feasible solution using Least Cost Method Problem

	Retail Agency					
Factories	1	2	3	4	5	Capacity
1	1	9	13	36	51	50
2	24	12	16	20	1	100
3	14	33	1	23	26	150
Requirement	100	60	50	50	40	300

Solution :
The Least Cost method is applied in the following manner, We observe that C_{11}= 1 is the minimum unit cost in the table. Hence X_{11}= 50 and the first row is crossed out since the row has no more capacity.

						Capacity
	1 **50**	9	13	36	51	~~50~~
	24	12 **60**	16	20	1 **40**	~~100~~ ~~60~~
	14 **50**	33	1 **50**	23 **50**	26	~~150~~ ~~100~~ ~~50~~
Requirement	~~100~~ ~~50~~	~~60~~	~~50~~	~~50~~	~~40~~	

Then the minimum unit cost in the uncrossed-out row and column is C_{25}=1, hence X_{25}=40 and the fifth column is crossed out.
Next C_{33}=1is the minimum unit cost, hence X_{33}=50 and the third column is crossed out.
Next C_{22}=12 is the minimum unit cost, henceX_{22}=60 and the second column is crossed out.
Next we look for the uncrossed-out row and column now C_{31}=14 is the minimum unit cost, hence X_{31}=50 and crossed out the first column since it was satisfied
Finally C_{34}=23 is the minimum unit cost, hence X_{34}=50 and the fourth column is crossed out.
So that the basic feasible solution developed by the LCM has
transportation cost is
1 x 50 + 12 x 60 + 1 x 40 + 14 x 50 + 1 x 50 + 23 x 50 = 2710
Note : That the minimum transportation cost obtained by the least cost method is much lower than the corresponding cost of the solution developed by using the North-West Corner Method.

Vogel Approximation Method (VAM):

VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1: For each row (column) with strictly positive capacity (requirement), determine a penalty by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

Step 2: Identify the row or column with the largest penalty among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal we select the topmost row and the extreme left column.

Step 3: We select X_{ij} as a basic variable if C_{ij} is the minimum cost in the row or column
With largest penalty. We choose the numerical value of X_{ij} as high as possible subject to the row and the column constraints. Depending upon whether ai or bj is the smaller of the two i_{th} row or j_{th} column is crossed out.

Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied.

Problem :
Solve the following transportation problem

Origin	Destination				a_i
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
b_j	60	40	30	110	240

Solution :
Note: a_i = capacity (supply) b_j = requirement (demand) Now, compute the penalty for various rows and columns which is shown in the following table:

Origin	Destination				a_i	Column Penalty
	1	2	3	4		
1	20	22	17	4	120	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

Look for the highest penalty in the row or column, the highest penalty occurs in the second column and the minimum unit cost i.e. C_{ij} in this column is C_{12}=22.
Hence assign 40 to this cell i.e. X_{12}=40 and cross out the second column (since second column was satisfied.

This is shown in the following table:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 **40**	17	4	80	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	

The next highest penalty in the uncrossed-out rows and columns is 13 which occur in the first row and the minimum unit cost in this row is C_{14} = 4, hence X_{14}= 80 and cross out the first row. The modified table is as follows

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 **40**	17	4 **80**	0	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

The next highest penalty in the uncrossed-out rows and columns is 8 which occurs in the third column and the minimum cost in this column is C_{23}=9, hence X_{23}=30 and cross out the third column with adjusted capacity, requirement and penalty values. The modified table is as follows

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 **40**	17	4 **80**	0	13
2	24	37	9 **30**	7	40	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the second row and the smallest cost in this row is C_{24}=15, hence X_{24}=30 and cross out the fourth column with the adjusted capacity, requirement and penalty values.

The modified table is as follows:

Origin	Destination 1	Destination 2	Destination 3	Destination 4	a_i	Column Penalty
1	20	22 **40**	17	4 **80**	0	13
2	24	37	9 **30**	7 **30**	10	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	

The transportation cost corresponding to this choice of basic variables is

22 x 40 + 4 x 80 + 9 x 30 + 7 x 30 + 24 x 10 + 32 x 50 = 3520

Modified Distribution Method :

The Modified Distribution Method, also known as MODI method or u-v method, which provides a minimum cost solution (optimal solution) to the transportation problem. The following are the steps involved in this method

Step 1: Find out the basic feasible solution of the transportation problem using any one of the three methods discussed in the previous section.

Step 2: Introduce dual variables corresponding to the row constraints and the column constraints. If there are m origins and n destinations then there will be m+n dual variables. The dual variables corresponding to the row constraints are represented by U_i, i = 1,2,…..m where as the dual variables corresponding to the column constraints are represented by V_j, j = 1,2,…..n. The values of the dual variables are calculated from the equation given below $U_i + V_j = C_{ij}$ if $X_{ij} > 0$

Step 3: Any basic feasible solution has m + n -1 and $X_{ij} > 0$. Thus, there will be m + n -1 equation to determine m + n dual variables. One of the dual variables can be chosen arbitrarily. It is also to be noted that as the primal constraints are equations, the dual variables are unrestricted in sign.

Step 4: If Xij =0, the dual variables calculated in Step 3 are compared with the C ij values of this allocation as $C_{ij} – U_i – V_j$. If al $C_{ij} – U_i – V_j \geq 0$, then by the *theorem of complementary slackness* it can be shown that the corresponding solution of the transportation problem is optimum. If one or more C_{ij} – Ui – Vj < 0, we select the cell with the least value of $C_{ij} – U_i – V_j$ and allocate as much as possible subject to the row and column constraints. The allocations of the number of adjacent cell are adjusted so that a basic variable becomes non-basic.

Step 5: A fresh set of dual variables are calculated and repeat the entire procedure from Step 1 to Step 5.

Problem 3:
Solve the following transportation problem using Modified Distribution method

						Supply
	1	9	13	36	51	**50**
	24	12	16	20	1	**100**
	14	33	1	23	26	**150**
Demand	**100**	**70**	**50**	**40**	**40**	**300**

Solution :-
Step 1: First we have to determine the basic feasible solution. The basic feasible solution using least cost method is

X_{11}=50, X_{22}=60, X_{25}=40, X_{31}=50, X_{32}=10, X_{33}=50 and X_{34}=40

Step 2: The dual variables U_1, U_2, U_3 and V_1, V_2, V_3, V_4, V_5 can be calculated from the corresponding C_{ij} values, that is

$U_1 + V_1 = 1$ $U_2 + V_2 = 12$ $U_2 + V_5 = 1$

$U_3 + V_1 = 14$ $U_3 + V_2 = 33$ $U_3 + V_3 = 1$ $U_3 + V_4 = 23$

Step 3: Choose one of the dual variables arbitrarily is zero that is u3=0 as it occurs most often in the above equations. The values of the variables calculated are
$U_1 = -13$, $U_2 = -21$, $U_3 = 0$ $V_1 = 14$, $V_2 = 33$, $V_3 = 1$, $V_4 = 23$, $V_5 = 22$

Step 4: Now we calculate $C_{ij} – U_i – V_j$ values for all the cells where X_{ij}=0 (i.e. un allocated cell by the basic feasible solution) That is
Cell (1,2) = $C_{12}-U_1-V_2$ = 9+13-33 = -11
Cell (1,3) = $C_{13}-U_1-V_3$ = 13+13-1 = 25
Cell (1,4) = $C_{14}-U_1-V_4$ = 36+13-23 = 26
Cell (1,5) = $C_{15}-U_1-V_5$ = 51+13-22 = 42
Cell (2,1) = $C_{21}-U_2-V_1$ = 24+21-14 = 31
Cell (2,3) = $C_{23}-U_2-V_3$ = 16+21-1 = 36
Cell (2,4) = $C_{24}-U_2-V_4$ = 20+21-23 = 18
Cell (3,5) = $C_{35}-U_3-V_5$ = 26-0-22 = 4l

Note that in the above calculation all the $C_{ij} – U_i – V_j \geq 0$ except for cell (1, 2)
where $C_{12} – U1 – V2$ = 9+13-33 = -11.
Thus in the next iteration x12 will be a basic variable changing one of the present basic variables non-basic. We also observe that for allocating one unit in cell (1, 2) we have to reduce one unit in cells (3, 2) and (1, 1)and increase one unit in cell (3, 1). The net transportation cost for each unit of such reallocation is -33 -1 + 9 +14 = -11
The maximum that can be allocated to cell (1, 2) is 10 otherwise the allocation in the cell (3, 2) will be negative. Thus, the revised basic feasible solution is

X_{11}= 40, X_{12}= 10, X_{22}= 60, X_{25}= 40, X_{31}= 60, X_{33}= 50, $X3_4$= 40

<u>Unbalanced Transportation Problem</u>:-

The total supply (capacity) at the origins is equal to the total demand (requirement) at the destination it is called balanced transportation problem

when the total supply is not equal to the total demand, which are called as unbalanced transportation problem.

In the unbalanced transportation problem if the total supply is more than the total demand then we introduce an additional column which will indicate the surplus supply with transportation cost zero.

Similarly, if the total demand is more than the total supply an additional row is introduced in the transportation table which indicates unsatisfied demand with zero transportation cost.

Problem :

Solve the following unbalanced transportation problem

Plant	W_1	W_2	W_3	Supply
X	20	17	25	400
Y	10	10	20	500
Demand	400	400	500	

Solution: -

In this problem the demand is 1300 whereas the total supply is 900. Thus, we now introduce an additional row with zero transportation cost denoting the unsatisfied demand. So that the modified transportation problem table is as follows>

	Warehouses			
Plant	W_1	W_2	W_3	Supply
X	20	17	25	400
Y	10	10	20	500
Unsatisfied Demand	0	0	0	400
Demand	400	400	500	1300

Now we can solve this problem as well as a balanced problem

Degenerate Transportation Problem :-

In a transportation problem, if a basic feasible solution with m origins and n destinations has less than m + n -1 positive X_{ij} i.e. occupied cells, then the problem is said to be a Degenerate transportation problem

The degeneracy problem does not cause any serious difficulty, but it can cause computational problem wile determining the optimal minimum solution.

Therefore it is important to identify a degenerate problem as early as beginning and take the necessary action to avoid any computational difficulty.

The degeneracy can be identified through the following results:

"In a transportation problem, a degenerate basic feasible solution exists if and only if some partial sum of supply (row) is equal to a partial sum of demand (column). For example the following transportation problem is degenerate. Because in this problem

$\mathbf{a_1 = 400 = b_1}$ $\mathbf{a_2 + a_3 = 900 = b_2 + b_3}$

Plant	Warehouses W1	W2	W3	Supply (a_i)
X	20	17	25	400
Y	10	10	20	500
Unsatisfied demand	0	0	0	400
Demand (b_j)	400	400	500	1300

There is a technique called perturbation, which helps to solve the degenerate problems.

Perturbation Technique: The degeneracy of the transportation problem can be avoided if we ensure that no partial sum of a_i (supply) and b_j (demand) is equal. We set up a new problem where

$a_i = a_i + d_i \quad i = 1, 2, \ldots\ldots, m$

$b_j = b_j \quad j = 1, 2, \ldots\ldots, n - 1$

$b_n = b_n + md \quad d > 0$

This modified problem is constructed in such a way that no partial sum of ai is equal to the bj. Once the problem is solved, we substitute d = 0 leading to optimum solution of the original problem

Example :-

Plant	Warehouses W1	W2	W3	Supply (a_i)
X	20	17	25	400 + d
Y	10	10	20	500 + d
Unsatisfied demand	0	0	0	400 + d
Demand (b_j)	400	400	500 + 3d	1300 + 3d

Now this modified problem can be solved by using any of the three methods viz. North-west Corner, or Least Cost, or VAM

ASSIGNMENT PROBLEM

Assignment Problem:-
Given n facilities, n jobs and the effectiveness of each facility to each job, here the problem is to assign each facility to one and only one job so that the measure of effectiveness if optimized. Here the optimization means Maximized or Minimized
There are many management problems has a assignment problem structure.

For example, the head of the department may have 6 people available for assignment and 6 jobs to fill. Here the head may like to know which job should be assigned to which person so that all tasks can be accomplished in the shortest time possible.

Another example a container company may have an empty container in each of the location 1, 2,3,4,5 and requires an empty container in each of the locations 6, 7, 8,9,10. It would like to ascertain the assignments of containers to various locations so as to minimize the total distance.

The third example here is, a marketing set up by making an estimate of sales performance for different salesmen as well as for different cities one could assign a particular salesman to a particular city with a view to maximize the overall sales.
Note that with n facilities and n jobs there are n! possible assignments.

The simplest wayof finding an optimum assignment is to write all the n! possible arrangements, evaluate their total cost and select the assignment with minimum cost. But this method leads to a Calculation problem of formidable size even when the value of n is moderate.

Assignment Problem Structure and Solution
The structure of the Assignment problem is similar to a transportation problem, is as follows:

		Jobs				
		1	2	...	n	
	1	c_{11}	c_{12}	...	c_{1n}	1
	2	c_{21}	c_{21}	...	c_{2n}	1
Workers		.	.	.	.	.
		.	.	.	.	
		.	.	.	.	
	n	c_{n1}	c_{n2}	...	c_{nn}	1
		1	1	...	1	

The element C_{ij} represents the measure of effectiveness when i^{th} person is assigned j^{th} job. Assume that the overall measure of effectiveness is to be minimized. The element X_{ij} represents the number of i^{th} individuals assigned to the j^{th} job. Since i^{th} the following

$X_{i1} + X_{i2} + \ldots\ldots + X_{in} = 1$, where i = 1, 2, ... , n person can be assigned only one job and j^{th} job can be assigned to only one person.

we have $X_{1j} + X_{2j} + \ldots\ldots + X_{nj} = 1$, where j = 1, 2, ... , n and the objective function is formulated as Minimize

$C_{11}X_{11} + C_{12}X_{12} + \ldots\ldots + C_{nn}X_{nn}$ and $X_{ij} \geq 0$

The assignment problem is actually a special case of the transportation problem where m = n and ai = bj = 1.
However, it may be easily noted that any basic feasible solution of an assignment problem contains (2n – 1) variables of which (n – 1) variables are zero.

Because of this high degree of degeneracy the usual computation techniques of a transportation problem become very inefficient. So, that a separate computation Technique is necessary for the assignment problem.

“ If a constant is added to every element of a row/column of the cost matrix of an Assignment problem the resulting assignment problem has the same optimum solution as the original assignment problem and vice versa”.

This result may be used in two different methods to solve the assignment problem. If in an assignment problem some cost elements Cij are negative, we may have to convert them into an equivalent assignment problem
where all the cost elements are non-negative by adding a suitable large constant to the cost elements of the relevant row or column, and then we look for a feasible solution which has zero assignment cost after adding suitable constants to the cost elements of the various rows and columns.
Since it has been assumed that all the cost elements are non-negative, this assignment must be optimum. On the basis of this principle a computational technique known as Hungarian Method is developed.

Hungarian Method: The Hungarian Method is discussed in the form of a series of computational steps as follows, when the objective function is that of minimization type.

Step 1: From the given problem, find out the cost table. Note that if the number of origins is not equal to the number of destinations then a dummy origin or destination must be added.

Step 2: In each row of the table find out the smallest cost element, subtract this smallest cost element from each element in that row. So, that there will be at least one zero in each row of the new table. This new table is known as First Reduced Cost Table.

Step 3: In each column of the table find out the smallest cost element, subtract this smallest cost element from each element in that column. As a result of this, each row and column has at least one zero element. This new table is known as Second Reduced Cost Table.

Step 4: Now determine an assignment as follows:
i. For each row or column with a single zero element cell that has not be assigned or eliminated, box that zero element as an assigned cell.
ii. For every zero that becomes assigned, cross out all other zeros in the same row and for column.
iii. If for a row and for a column there are two or more zero and one can’t be chosen by inspection, choose the assigned zero cell arbitrarily.
iv. The above procedures may be repeated until every zero element cell is either assigned(boxed) or crossed out.

Step 5: An optimum assignment is found, if the number of assigned cells is equal to the number of rows (and columns). In case we had chosen a zero cell arbitrarily, there may be an alternate optimum. If no optimum solution is found i.e. some rows or columns without an assignment then go to Step 6.

Step 6:Draw a set of lines equal to the number of assignments which has been made in Step 4, covering all the zeros in the following manner

- **i.** Mark check (√) to those rows where no assignment has been made
- **ii.** Examine the checked (√) rows. If any zero element cell occurs in those rows, check (√) the respective columns that contains those zeros
- **iii.** Examine the checked (√) columns. If any assigned zero element occurs in those columns, check (√) the respective rows that contain those assigned zeros.
- **iv.** The process may be repeated until now more rows or column can be checked.
- **v.** Draw lines through all unchecked rows and through all checked columns.

Step 7 : Examine those elements that are not covered by a line. Choose the smallest of these elements and subtract this smallest from all the elements that do not have a line through them, Add this smallest element to every element that lies at the intersection of two lines. Then the resulting matrix is a new revised cost table

.

Problem1:

A work shop contains four persons available for work on the four jobs. Only one person can work on any one job. The following table shows the cost of assigning each person to each job. The objective is to assign person to jobs such that the total assignment cost is a minimum.

		Jobs 1	**2**	**3**	**4**
Persons	**A**	20	25	22	28
	B	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

Solution :

Step 1: From the given problem, find out the cost table. Note that if the number of origins is not equal to the number of destinations then a dummy origin or destination must be added.

Problem has 4 row and 4 columns. So it is balanced problem

Step 2: In each row of the table find out the smallest cost element, subtract this smallest cost element from each element in that row. So, that there will be at least one zero in each row of the new table. This new table is known as First Reduced Cost Table.

0	**5**	**2**	**8**
0	**3**	**8**	**2**
2	**0**	**4**	**7**
2	**0**	**1**	**1**

Step 3: In each column of the table find out the smallest cost element, subtract this smallest cost element from each element in that column. As a result of this, each row and column has at least one zero element. This new table is known as Second Reduced Cost Table.

		Jobs 1	2	3	4
Persons	A	0	5	1	7
	B	0	3	7	1
	C	2	0	3	6
	D	2	0	0	0

Step 4: Determine an Assignment

By examine row A of the table in Step 3, we find that it has only one zero (cell A_1) box this zero and cross out all other zeros in the boxed column. In this way we can eliminate cell B_1.Now examine row C, we find that it has one zero (cell C_2) box this zero and cross out (eliminate) the zeros in the boxed column. This is how cell D_2 gets eliminated. There is one zero in the column 3. Therefore, cell D_3 gets boxed and this enables us to eliminate cell D_4.Therefore, we can box (assign) or cross out (eliminate) all zero's. The resultant table is shown below

		Jobs 1	2	3	4
Persons	A	[0]	5	1	7
	B	0	3	7	1
	C	X	[0]	3	6
	D	2	X	[0]	X

Step 5: The solution obtained in Step 4 is not optimal. Because we were able to make three assignments when four were required

Step 6: Cover all the zeros of the table shown in the Step 4 with three lines (since already we made three assignments).Check row B since it has no assignment. Note that row B has a zero in column 1, therefore check column 1.
Then we check row A since it has a zero in column 1. Note that no other rows and columns are checked. Now we may draw three lines through unchecked rows (row C and D) and the checked column(column 1). This is shown in the table given below**:**

		Jobs			
		1	2	3	4
Persons	A	[0]	5	1	7
	B	0	3	7	1
	C	2	[0]	3	6
	D	2	0	0	0

Step 7: Develop the new revised table.
Examine those elements that are not covered by a line in the table given in Step 6.
Take the smallest element in this case the smallest element is 1. Subtract this smallest element from the uncovered cells and add 1 to elements (C_1 and D_1) that lie at the intersection of two lines.
Finally, we get the new revised cost table, which is shown below

		Jobs			
		1	**2**	**3**	**4**
Persons	**A**	0	4	0	6
	B	0	2	6	0
	C	3	0	3	6
	D	3	0	0	0

Step 8: Now, go to Step 4 and repeat the procedure until we arrive at an optimal solution (assignment).
Step 9: Determine an assignment Examine each of the four rows in the table given in Step 7, we may find that it is only row C which has only one zero box this cell C_2 and cross out D_2.

Note that all the remaining rows and columns have two zeros. Choose a zero arbitrarily, say A_1 and box this cell so that the cells A_3 and B_1 get eliminated.
Now row B (cell B_4) and column 3 (cell D_4) has one zero box these cells so that cell D_4 is eliminated. Thus, all the zeros are either boxed or eliminated. This is shown in the following table

		Jobs 1	2	3	4
Persons	A	[0]	4	~~0~~	6
	B	0	2	6	[0]
	C	~~3~~	[0]	3	6
	D	3	~~0~~	[0]	~~0~~

Since the number of assignments equal to the number of rows (columns), the assignment shown in the above tale is optimal.
The total cost of assignment is: 78
that is **A1 + B4 + C2 + D3 = 20 + 17 + 17 + 24 = 78**

Unbalanced Assignment Problem:

In the previous section we assumed that the number of persons to be assigned and the number of jobs were same. Such kind of assignment problem is called as balanced assignment problem.
Suppose if the number of person is different from the number of jobs then the assignment problem is called as unbalanced.
If the number of jobs is less than the number of persons, some of them can't be assigned any job. So that we have to introduce on or more dummy jobs of zero duration to make the unbalanced assignment problem into balanced assignment problem.
This balanced assignment problem can be solved by usingthe Hungarian Method as discussed in the previous section. The persons to whom the dummy jobs are assigned are left out of assignment. Similarly, if the number of persons is less than number of jobs then we have introduce one or more dummy persons with zero duration to modify the unbalanced into balanced and then the problem is solved using the Hungarian Method. Here the jobs assigned to the dummy persons are left out.

Problem :
Solve the following unbalanced assignment problem of minimizing the total time for performing all the jobs

		Jobs 1	2	3	4	5
Workers	A	5	2	4	2	5
	B	2	4	7	6	6
	C	6	7	5	8	7
	D	5	2	3	3	4
	E	8	3	7	8	6
	F	3	6	3	5	7

Solution :

In this problem the number of jobs is less than the number of workers so we have to introduce a dummy job with zero duration. The revised assignment problem is as follows:

		Jobs 1	2	3	4	5	6
Workers	A	5	2	4	2	5	0
	B	2	4	7	6	6	0
	C	6	7	5	8	7	0
	D	5	2	3	3	4	0
	E	8	3	7	8	6	0
	F	3	6	3	5	7	0

Now the problem becomes balanced one since the number of workers is equal to the number jobs. So that the problem can be solved using Hungarian Method.

Step 1: The cost table

Workers		Jobs 1	2	3	4	5	6
	A	5	2	4	2	5	0
Workers	B	2	4	7	6	6	0
	C	6	7	5	8	7	0
	D	5	2	3	3	4	0
	E	8	3	7	8	6	0
	F	3	6	3	5	7	0

Step 2: Find the First Reduced Cost Table

Workers		Jobs 1	2	3	4	5	6
	A	5	2	4	2	5	0
	B	2	4	7	6	6	0
Workers	C	6	7	5	8	7	0
	D	5	2	3	3	4	0
	E	8	3	7	8	6	0
	F	3	6	3	5	7	0

Step 3 Find the second Cost Reduced table:

Workers		Jobs 1	2	3	4	5	6
	A	3	0	1	0	1	0
	B	0	2	4	4	2	0
Workers	C	4	5	2	6	3	0
	D	3	0	0	1	0	0
	E	6	1	4	6	2	0
	F	1	4	0	3	3	0

Step 4 : Determine an Assignment By using the Hungarian Method the assignment is made as follows

Jobs

Workers		1	2	3	4	5	6
	A	3	~~0~~	1	[0]	1	~~0~~
	B		2	4	4	2	0
Workers	C	[0]	5	2	6	3	X
	D	3	~~0~~	~~0~~	1	[0]	~~0~~
	E	6	1	4	6	2	[0]
	F	1	4		3	3	0

Step 5 : The solution obtained in Step 4 is not optimal. Because we were able to make five assignments when six were required.

Step 6: Cover all the zeros of the table shown in the Step 4 with five lines (since already we made five assignments).

Step 7: Develop the new revised table. Examine those elements that are not covered by a line in the table given in Step 6. Take the smallest element in this case the smallest element is 1. Subtract this smallest element from the uncovered cells and add 1 to elements (A_6, B_6, D_6 and F_6) that lie at the intersection of two lines. Finally, we get the new revised cost table, which is shown below

Jobs

Workers		1	2	3	4	5	6
	A	3	0	1	0	1	1
	B	0	2	4	4	2	1
Workers	C	3	4	1	5	2	0
	D	3	0	0	1	0	1
	E	5	0	3	5	1	0
	F	1	4	0	3	3	1

Step 8: Now, go to Step 4 and repeat the procedure until we arrive at an optimal solution (assignment).

Step 9: Determine an assignment

		Jobs					
		1	2	3	4	5	6
Workers	A	3	~~0~~	1	[0]	1	1
	B	[0]	2	4	4	2	1
	C	~~0~~	4	1	5	2	[0]
	D	3	~~0~~	~~0~~	1	[0]	1
	E	5	[0]	3	5	1	0
	F	1	4	[0]	3	3	1

Since the number of assignments equal to the number of rows (columns), the assignment shown in the above tale is optimal.

Thus, the worker A is assigned to Job4, worker B is assigned to job 1, worker C is assigned To job 6, worker D is assigned to job 5, worker E is assigned to job 2, and worker F is assigned to job 3. Since the Job 6 is dummy so that worker C can't be assigned. The total Minimum time is 14, that is $\mathbf{A_4 + B_1 + D_5 + E_2 + F_3 = 2 + 2 + 4 + 3 + 3 = 14}$

Infeasible Assignment Problem :

Sometimes it is possible a particular person is incapable of performing certain job or a specific job can't be performed on a particular machine. In this case the solution of the problem takes into account of these restrictions so that the infeasible assignment can be avoided.

The infeasible assignment can be avoided by assigning a very high cost to the cells where assignments are restricted or prohibited.

Problem :

A computer centre has five jobs to be done and has five computer machines to perform them. The cost of processing of each job on any machine is shown in the table below.

		Jobs				
		1	2	3	4	5
Computer Machines	1	70	30	X	60	30
	2	X	70	50	30	30
	3	60	X	50	70	60
	4	60	70	20	40	X
	5	30	30	40	X	70

Because of specific job requirement and machine configurations certain jobs can't be done on certain machines. These have been shown by X in the cost table. The assignment of jobs to the machines must be done on a one to one basis. The objective here is to assign the jobs to the available machines so as to minimize the total cost without violating the restrictions as mentioned above.

Solution :-

Step 1: The cost Table; Because certain jobs cannot be done on certain machines we assign a high cost say for example500 to these cells i.e. cells with X and modify the cost table. The revised assignment problem is as follows**:**

		Jobs				
		1	2	3	4	5
Computer Machines	1	70	30	500	60	30
	2	500	70	50	30	30
	3	60	500	50	70	60
	4	60	70	20	40	500
	5	30	30	40	500	70

Now we can Solve this Problem Using Hungarian Method

Step 2: Find the First Reduced Cost Table

		Jobs				
		1	2	3	4	5
Computer Machines	1	40	0	470	30	0
	2	470	40	20	0	0
	3	10	450	0	20	10
	4	40	50	0	20	480
	5	0	0	10	470	40

Step 3: Find the Second Reduced Cost Table

Computer Machines	Jobs 1	2	3	4	5
1	40	0	470	30	0
2	470	40	20	0	0
3	10	450	0	20	10
4	40	50	0	20	480
5	0	0	10	470	40

Step 4: Determine an Assignment

Computer Machines	Jobs 1	2	3	4	5
1	40	[0]	470	30	~~0~~
2	470	40	20	0	0
3	10	450	0	[0]	10
4	40	50	~~0~~	20	480
5	[0]	~~0~~	[0]	470	40

Step 5: The solution obtained in Step 4 is not optimal. Because we were able to make four assignments when five were required.

Step 6: Cover all the zeros of the table shown in the Step 4 with four lines (since already we made four assignments).
Check row 4 since it has no assignment. Note that row 4 has a zero in column 3, therefore check column3. Then we check row 3 since it has a zero in column 3. Note that no other rows and columns are checked. Now we may draw four lines through unchecked rows (row 1, 2, 3 and 5) and the checked column (column 3). This is shown in the table given below

		Jobs 1	2	3	4	5
Computer Machines	1	40	0	470	30	0
	2	470	40	20		0
	3	10	450		0	10
	4	40	50	0	20	480
	5	0	0	0	470	40

Step 7: Develop the new revised table. Examine those elements that are not covered by a line in the table given in Step 6. Take the smallest element in this case the smallest element is 10. Subtract this smallest element from the uncovered cells and add 1 to elements (A_6, B_6, D_6 and F_6) that lie at the intersection of two lines. Finally, we get the new revised cost table, which is shown below

		Jobs 1	2	3	4	5
Computer Machines	1	40	0	471	30	0
	2	470	40	21	0	0
	3	0	440	0	10	0
	4	30	40	0	10	470
	5	0	0	11	470	40

Step 8: Now, go to Step 4 and repeat the procedure until we arrive at an optimal solution (assignment).

Step 9: Determine an assignment

Since the number of assignments equal to the number of rows (columns), the assignment shown in the above tale is optimal.

Thus, the Machine1 is assigned to Job5, Machine 2 is assigned to job4, Machine3 is assigned tojob1, Machine 4 is assigned to job3 and Machine5 is assigned to job2.

The minimum assignment cost is: 170

Traveling Salesman problem:

Traveling salesman problem is similar to the assignment problem, but here two extra restrictions are imposed. The first restriction is that we cannot select the element in the leading diagonal as we do not follows **i** again by **i .**The second restriction is that we do not produce an item again until all the items are produced once. The second restriction means no city is visited twice until the tour of all the cities is completed

Mathematically a traveling salesman problem can be stated as follows: Optimize

$$\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}$$

subject to

$$\left.\begin{array}{l} \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,\dots,n \\ \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,\dots,n \\ x_{ij} = 0 \ \ or \ \ 1, \ \ i = 1,\dots,n \ \ , \ \ j = 1,\dots n. \\ The\ first\ and\ second\ restriction. \end{array}\right\}$$

Where d_{ij} is the distance from city i to city j, and x_{ij} is to be some positive integer or zero, and the only possible integer is one, so the condition of $x_{ij} = 0$ or 1, is automatically satisfied.

Associated to each traveling salesman problem there is a matrix called distance matrix $[d_{ij}]$ where d_{ij} is the distance from city i to city j. In this paper we call it distance matrix, and represent it as follows:

$$\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & \cdots & n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1n} \\ d_{11} & d_{22} & d_{23} & \cdots & d_{2n} \\ \vdots & & & & \\ d_{n1} & d_{n2} & d_{n3} & \cdots & d_{nn} \end{bmatrix} \end{array}$$

which is always a square matrix, thus each city can be assigned to only one city.
In fact any solution of this problem will contain exactly m non-zero positive individual allocations.

Steps to Solve Travelling Sales Man Problems:

Step 1

In a minimization (maximization) case, find the minimum (maximum) element of each row in the distance matrix (say a_i) and write it on the right hand side of the matrix.

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{1n} \\ d_{11} & d_{22} & d_{23} & \cdots & d_{2n} \\ \vdots \\ d_{n1} & d_{n2} & d_{n3} & \cdots & d_{nn} \end{bmatrix} \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{matrix}$$

Then divide each element of i th row of the matrix by a_i . These operations create at least one ones in each rows.

$$\begin{bmatrix} d_{11}/a_1 & d_{12}/a_1 & d_{13}/a_1 & \cdots & d_{1n}/a_1 \\ d_{11}/a_2 & d_{22}/a_2 & d_{23}/a_2 & \cdots & d_{2n}/a_2 \\ \vdots \\ d_{n1}/a_n & d_{n2}/a_n & d_{n3}/a_n & \cdots & d_{nn}/a_n \end{bmatrix} \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{matrix}$$

In term of ones for each row and column do assignment, otherwise go to step 2 .

Step 2

Find the minimum (maximum) element of each column in distance matrix (say b_j), and write it below j th column. Then divide each element of j th column of the matrix by b_j .

These operations create at least a one in each column. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step 3.

$$\begin{matrix} \begin{bmatrix} d_{11}/a_1b_1 & d_{12}/a_1b_2 & d_{13}/a_1b_3 & \cdots & d_{1n}/a_1b_n \\ d_{11}/a_2b_1 & d_{22}/a_2b_2 & d_{23}/a_2b_3 & \cdots & d_{2n}/a_2b_n \\ \vdots \\ d_{n1}/a_nb_1 & d_{n2}/a_nb_2 & d_{n3}/a_nb_3 & \cdots & d_{nn}/a_nb_n \end{bmatrix} & \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{matrix} \\ \begin{matrix} b_1 & \quad b_2 & \quad b_3 & \quad \cdots & \quad b_n \end{matrix} & \end{matrix}$$

Note: In a maximization case, the end of step 2 we have a fuzzy matrix, which all elements are belong to $[0,1]$, and the greatest element is one [6].

Step 3

Draw the minimum number of lines to cover all the ones of the matrix. If the number of drowned lines less than n , then the complete solution is not possible, while if the number of lines is exactly equal to n , then the complete solution is obtained.

Step 4

If a complete solution is not possible in step 3, then select the smallest (largest) element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation create some new ones to this row or column.

If still a complete optimal solution is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal solution will be obtained.

Priority plays an important role in this method, when we want to assign the ones.

Priority rule:

For maximization (minimization) traveling salesman problem, assign the ones on the rows which have greatest (smallest) element on the right hand side, respectively.

If a tour is not reached, so do the assignment that will make a tour. We note that if a tour does not occur, then assign the element immediately greater than one

One question arise here, what to do with non square matrix? To make square, a non square matrix, we add one artificial row or column which all elements are one. Thus we solve the problem with the new matrix, by using the new method.

The matrix after performing the steps reduces to a matrix which has ones in each row and each column. So, the optimal solution has been reached

Problem:

Consider the following traveling salesman problem. Design a tour to five cities to the salesman such that minimize the total distance. Distance between cities is shown in the following matrix.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} - & 10 & 3 & 6 & 9 \\ 5 & - & 5 & 4 & 2 \\ 4 & 9 & - & 7 & 8 \\ 7 & 1 & 3 & - & 4 \\ 3 & 2 & 6 & 5 & - \end{bmatrix} \end{array}$$

Solution :

Find the minimum element of each row in the distance matrix (say a_i) and write it on the right hand side of the matrix, as follows:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} - & 10 & 3 & 6 & 9 \\ 5 & - & 5 & 4 & 2 \\ 4 & 9 & - & 7 & 8 \\ 7 & 1 & 3 & - & 4 \\ 3 & 2 & 6 & 5 & - \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 2 \\ 4 \\ 1 \\ 2 \end{array}$$

Then divide each element of i th row of the matrix by a_i. These operations create some ones to each row, and the matrix reduces to following matrix.

$$
\begin{bmatrix} - & 3.3 & 1 & 2 & 3 \\ 2.5 & - & 2.5 & 2 & 1 \\ 1 & 2.25 & - & 1.75 & 2 \\ 7 & 1 & 3 & - & 4 \\ 1.5 & 1 & 3 & 2.5 & - \end{bmatrix} \begin{matrix} 3 \\ 2 \\ 4 \\ 1 \\ 2 \end{matrix}
$$

Now find the minimum element of each column in distance matrix (say b_j), and divide each element of j th column of the matrix by b_j .This operation create some ones to each row and each column. This operation create some ones to each row and each column

$$
\begin{bmatrix} - & 3.3 & 1 & 1.14 & 3 \\ 2.5 & - & 2.5 & 1.14 & 1 \\ 1 & 2.25 & - & 1 & 2 \\ 7 & 1 & 3 & - & 4 \\ 1.5 & 1 & 3 & 1.428 & - \end{bmatrix} \begin{matrix} 3 \\ 2 \\ 4 \\ 1 \\ 2 \end{matrix}
$$

The minimum number of lines required to pass through all ones is 4, and the minimum element of the uncovered is 1.428 on 5th row, so divide each element of 5th row of the matrix by 1.428.

$$
\begin{bmatrix} - & 3.3 & 1 & 1.14 & 3 \\ 2.5 & - & 2.5 & 1.14 & 1 \\ 1 & 2.25 & - & 1 & 2 \\ 7 & 1 & 3 & - & 4 \\ 1.05 & 0.7 & 2.1 & 1 & - \end{bmatrix} \begin{matrix} 3 \\ 2 \\ 4 \\ 1 \\ 2 \end{matrix}
$$

Now, minimum number of lines required to pass through all the ones of the matrix is 5. So, the complete solution is possible, and we can assign the ones, it is based on priority rule. Priority rule is assigning one on the rows which have the smallest element on the right hand side, respectively.

The details of this program are as follows:

City 1 assigns to City 3 distance 3
City 2 assigns to City 5 distance 2
City 3 assigns to City 4 distance 7
City 4 assigns to City 2 distance 1
City 5 assigns to City 1 distance 3

so the optimal assignment has been reached, and the optimal path is $(1,3),(3,4),(4,2),(2,5),(5,1)$ and total distance according to this plan is 16.

Problem : Consider the following traveling salesman problem. Design a tour to five cities to the salesman such that minimize the total distance. Distance between cities is shown in the following matrix.

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix} - & 11 & 10 & 12 & 4 \\ 2 & - & 6 & 3 & 5 \\ 3 & 12 & - & 14 & 6 \\ 6 & 14 & 4 & - & 7 \\ 7 & 9 & 8 & 12 & - \end{bmatrix} \end{array}
$$

Now the minimum element of second column is 1.28 . Divide each element of second column by 1.28 and the minimum element of 4th column is 1.5 . Divide each element of 4th column by 1.5 . These operations create some ones on second and 4th column, and the reduced matrix is as follows:

$$
\begin{bmatrix} - & 2.148 & 2.5 & 2 & 1 \\ 1 & - & 3 & 1 & 2.5 \\ 1 & 3.125 & - & 2.25 & 2 \\ 1.5 & 2.734 & 1 & - & 1.75 \\ 1 & 1 & 1.14 & 1.14 & - \end{bmatrix}
\begin{array}{c} 4 \\ 2 \\ 3 \\ 4 \\ 7 \end{array}
$$

The minimum number of lines required to pass through all the ones of the matrix is 5.

So, the complete solution is possible, and we can assign the ones, it is based on priority rule. Priority rule is assigning one on the rows which have the smallest element on the right hand side, respectively.

The details of this program are as follows:

City 1 assigns to City 5 distance 4
City 2 assigns to City 4 distance 3
City 3 assigns to City 1 distance 3
City 4 assigns to City 3 distance 4
City 5 assigns to City 2 distance 9.

So the optimal path has been reached, $2-4-3-1-5-2$, and total distance according to this plan is 23.

Tutorial Questions

1. A company has factories at F1, F2 and F3 that supply products to ware houses at W1, W2 and W3 .The weekly capacities of the factories are 200,160 and 90 units. The weekly warehouse requirements are 180,120 and150/units respectively. The unit shipping costs in rupees are as follows find the optimal solution

	W1	W2	W3	supply
F1	16	20	12	200
F2	14	8	18	160
F3	26	24	16	90
Demand	180	120	150	

2. Different machines can do any of the five required jobs with different profits ring from each assignment as shown in adjusting table. Find out maximum profit possible through optimal assignment

Jobs	Machines				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

3. a). Briefly explain about the assignment problems in OR and applications of assignment in OR?

   b) What do you understand by degeneracy in a transportation problem?

4. a) Give the mathematical formulation of Transportation problem

   b) Use Vogel's approximate method to obtain an initial basic feasible solution of a transportation problem and find the optimal solution

Warehouse / Factory	W	X	Y	Z	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

Assignment Questions

1. Six jobs go first on machine A, then on machine B and last on a machine C. The order of completion of jobs have no significance. The following table gives machine time for the six jobs and the three machines. Find the sequence of jobs that minimizes elapsed time to complete the jobs.

Jobs	Processing Time		
	Machine A	Machine B	Machine C
1	8	3	8
2	3	4	7
3	7	5	6
4	2	2	9
5	5	1	10
6	1	6	9

2 a) What do you understand by degeneracy in a transportation problem?

b) Obtain initial solution in the following transportation problem by using VAM and LCM

Source	D1	D2	D3	D4	D5	Availability
S1	5	3	8	6	6	1100
S2	4	5	7	6	7	900
S3	8	4	4	6	6	700
Requirement	800	400	500	400	600	

3. Different machines can do any of the five required jobs with different profits resulting from each assignment as shown in the adjusting table. Find out maximum profit possible through optimalassignment.

Jobs	Machines				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

4. a) State the assignment problem mathematically.

b) For the assignment table, find the assignment of salesmen to districts that will result in maximum sales

Districts / Sales people	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

5. a) What do you understand by degeneracy in a transportation problem?

b)A company has three plants at locations A.B.C which supply to Warehouse located at D,E,F,G and
6. H. Monthly plant capacities are 800,500, and 900 respectively. Monthly warehouse requirements are 400,500,400 and 800units.Unit Transportation cost in rupees is

	D	E	F	G	H
A	5	8	6	6	3
B	4	7	7	6	5
C	8	4	6	6	4

7. Determine the optimum distribution for the company in order to minimize total transportation cost by NWCR

UNIT 3

Theory of Games

UNIT III
Theory of Games

Introduction : The definition given by William G. Nelson runs as follows: "Game theory, more properly the theory of games of strategy, is a mathematical method of analyzing a conflict. The alternative is not between this decision or that decision, but between this strategy or that strategy to be used against the conflicting interest".

In the perception of Robert Mockler, "Game theory is a mathematical technique helpful in making decisions in situations of conflicts, where the success of one part depends at the expense of others, and where the individual decision maker is not in complete control of the factors influencing the outcome".

According to von Neumann and Morgenstern, "The 'Game' is simply the totality of the rules which describe it. Every particular instance at which the game is played – in a particular way – from beginning to end is a 'play'. The game consists of a sequence of moves, and the play of a sequence of choices".

According to Edwin Mansfield, "A game is a competitive situation where two or more persons pursue their own interests and no person can dictate the outcome. Each player, an entity with the same interests, make his own decisions. A player can be an individual or a group".

Assumptions for a Competitive Game:

Game theory helps in finding out the best course of action for a firm in view of the anticipated countermoves from the competing organizations. A competitive situation is a competitive game if the following properties hold,

1. The number of competitors is finite, say N.
2. A finite set of possible courses of action is available to each of the N competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that all the players select their courses of action simultaneously. As a result, nocompetitor will be in a position to know the choices of his competitors.
4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

Managerial Applications of the Theory of Games:

The techniques of game theory can be effectively applied to various managerial problems as detailed below:

1) Analysis of the market strategies of a business organization in the long run.
2) Evaluation of the responses of the consumers to a new product.
3) Resolving the conflict between two groups in a business organization.
4) Decision making on the techniques to increase market share.
5) Material procurement process.
6) Decision making for transportation problem.
7) Evaluation of the distribution system.

8) Evaluation of the location of the facilities.
9) Examination of new business ventures and
10) Competitive economic environment.

<u>Concepts in the Theory of Games:</u>

Players: The competitors or decision makers in a game are called the players of the game.

Strategies: The alternative courses of action available to a player are referred to as his strategies.

Pay off: The outcome of playing a game is called the pay off to the concerned player.

Optimal Strategy: A strategy by which a player can achieve the best pay off is called the optimal strategy for him.

Zero-sum game: A game in which the total payoffs to all the players at the end of the game is zero is referred to as a zero-sum game.

Non-zero sum game: Games with “less than complete conflict of interest” are called non-zero sum games. The problems faced by a large number of business organizations come under this category. In such games, the gain of one player in terms of his success need not be completely at the expense of the other player.

Payoff matrix: The tabular display of the payoffs to players under various alternatives is called the payoff matrix of the game.

Pure strategy: If the game is such that each player can identify one and only one strategy as the optimal strategy in each play of the game, then that strategy is referred to as the best strategy for that Player and the game is referred to as a game of pure strategy or a pure game
.
Mixed strategy: If there is no one specific strategy as the ‘best strategy’ for any player in a game, then the game is referred to as a game of mixed strategy or a mixed game. In such a game, each player has to choose different alternative courses of action from time to time.

N-person game: A game in which N-players take part is called an N-person game.

Maxi min-Mini max Principle: The maximum of the minimum gains is called the maxi min value of the game and the corresponding strategy is called the maxi min strategy. Similarly the minimum of the maximum losses is called the mini max value of the game and the corresponding strategy is called the mini max strategy. If both the values are equal, then that would guarantee the best of the worst results.

Negotiable or cooperative game: If the game is such that the players are taken to cooperate on any or every action which may increase the payoff of either player, then we call it a negotiable or cooperative game.

Non-negotiable or non-cooperative game: If the players are not permitted for coalition then we refer to the game as a non-negotiable or non-cooperative game.

Saddle point: A saddle point of a game is that place in the payoff matrix where the maximum of the row minima is equal to the minimum of the column maxima. The payoff at the saddle point is called **the value of the game** and the corresponding strategies are called the **pure strategies.**

Dominance: One of the strategies of either player may be inferior to at least one of the remaining ones. The superior strategies are said to dominate the inferior ones.

Types of Games:

There are several classifications of a game. The classification may be based on various factors such as the number of participants, the gain or loss to each participant, the number of Strategies available to each participant, etc. Some of the important types of games are enumerated below.

Two person games and n-person games : In two person games, there are exactly two players and each competitor will have a finite number of strategies. If the number of players in a game exceeds two, then we refer to the game as n-person game.

Zero sum game and non-zero sum game: If the sum of the payments to all the players in a game is zero for every possible outcome of the game, then we refer to the game as a zero sum game. If the sum of the payoffs from any play of the game is either positive or negative but not zero, then the game is called a non-zero sum game

Games of perfect information and games of imperfect information: A game of perfect information is the one in which each player can find out the strategy that would be followed by his opponent. On the other hand, a game of imperfect information is the one in which no player can know in advance what strategy would be adopted by the competitor and a player has to proceed in his game with his guess works only.

Games with finite number of moves / players and games with unlimited number of moves : A game with a finite number of moves is the one in which the number of moves for each player is limited before the start of the play. On the other hand, if the game can be continued over an extended period of time and the number of moves for any player has no restriction, then we call it a game with unlimited number of moves.

Constant-sum games: If the sum of the game is not zero but the sum of the payoffs to both players in each case is constant, then we call it a constant sum game. It is possible to reduce such a game to a zero sum game.

2x2 two person game and 2xn and mx2 games: When the number of players in a game is two and each player has exactly two strategies, the game is referred to as 2x2 two person game. A game in which the first player has precisely two strategies and the second player has three or more strategies is called an 2xn game. A game in which the first player has three or more strategies and the second player has exactly two strategies is called an mx2 game.

3x3 and large games: When the number of players in a game is two and each player has exactly three strategies, we call it a 3x3 two person game. Two-person zero sum games are said to be larger if each of the two players has 3 or more choices. The examination of 3x3 and larger games is involves difficulties. For such games, the technique of linear programming can be used as a method of solution to identify the optimum strategies for the two players.

Non-constant games: Consider a game with two players. If the sum of the payoffs to the two players is not constraint in all the plays of the game, then we call it a non-constant game. Such games are divided into negotiable or cooperative games and non-negotiable or non-cooperative games.
Two-person zero sum games: A game with only two players, say player A and player B, is called a two-person zero sum game if the gain of the player A is equal to the loss of the player B, so that the total sum is zero.

Payoff matrix: When players select their particular strategies, the payoffs (gains or losses) can be represented in the form of a payoff matrix..Since the game is zero sum, the gain of one player is equal to the loss of other and vice-versa. Suppose A has m strategies and B has n strategies. Consider the following payoff matrix. Player A wishes to gain as large a payoff **a**ij as possible while player B will do his best to reach as small a value ij a as possible where the gain to player B and loss to player A be (-**a**ij).
The amount of payoff, i.e., V at an equilibrium point is known as the **value of the game**. The optimal strategies can be identified by the players in the long run.

Fair game: The game is said to be fair if the value of the game V = 0.

Player B's strategies

Player A's strategies

$$\begin{array}{c} \\ A_1 \\ A_1 \\ \vdots \\ Am \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

Assumptions for two-person zero sum game:

For building any model, certain reasonable assumptions are quite necessary. Some assumptions for building a model of two-person zero sum game are listed below.

a) Each player has available to him a finite number of possible courses of action. Sometimes the set of courses of action may be the same for each player. Or, certain courses of action may be available to both players while each player may have certain specific courses of action which are not available to the other player.

b) Player A attempts to maximize gains to himself. Player B tries to minimize losses to himself.

c) The decisions of both players are made individually prior to the play with no communication between them.

d) The decisions are made and announced simultaneously so that neither player has an advantage resulting from direct knowledge of the other player's decision.

e) Both players know the possible payoffs of themselves and their opponents. Mini max and Maxi min Principles

The selection of an optimal strategy by each player without the knowledge of the competitor's strategy is the basic problem of playing games. The objective of game theory is to know how these players must select their respective strategies, so that they may optimize their payoffs. Such a criterion of decision making is referred to as mini max-maxi min principle. This principle in games of pure strategies leads to the best possible selection of a strategy for both players.

For example, if player A chooses his **ith** strategy, then he gains at least the payoff min **a ij** , which is minimum of the **ith** row elements in the payoff matrix. Since his objective is to Maximize his payoff, he can choose strategy i so as to make his payoff as large as possible. i.e., a payoff which is not less than

$$\max_{1 \le i \le m} \min_{1 \le j \le n} a_{ij}$$

Similarly player B can choose jth column elements so as to make his loss not greater than

$$\min_{1 \le j \le n} \max_{1 \le i \le m} a_{ij} .$$

If the maxi min value for a player is equal to the mini max value for another player, i.e.

$$\max_{1 \le i \le m} \min_{1 \le j \le n} a_{ij} = V = \min_{1 \le j \le n} \max_{1 \le i \le m} a_{ij}$$

then the game is said to have a saddle point (equilibrium point) and the corresponding strategies are called optimal strategies. If there are two or more saddle points, they must be equal.

Problem:

Solve the game with the following pay-off matrix.

Player B

Strategies

Player A Strategies		I	II	III	IV	V
	1	–2	5	– 3	6	7
	2	4	6	8	–1	6
	3	8	2	3	5	4
	4	15	14	18	12	20

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-3
2	-1
3	2
4	12

Maximum of {-3, -1, 2, 12} = 12

Next consider the maximum of each column.

Column	Maximum Value
1	15
2	14
3	18
4	12
5	20

Minimum of {15, 14, 18, 12, 20}= 12

We see that the maximum of row minima = the minimum of the column maxima. So the

game has a saddle point. The common value is 12. Therefore the value V of the game = 12.
Interpretation: In the long run, the following best strategies will be identified by the two players

The best strategy for player A is strategy 4.
The best strategy for player B is strategy IV.
The game is favorable to player A.

Problem 2: Solve the game with the following pay-off matrix

Strategies

Player X Strategies	*I*	*II*	*III*	*IV*	*V*
1	9	12	7	14	26
2	25	35	20	28	30
3	7	6	–8	3	2
4	8	11	13	–2	1

Solution: First consider the minimum of each row.

Row	Minimum Value
1	7
2	20
3	-8
4	-2

Maximum of {7, 20, –8, -2} = 20

Next consider the maximum of each column.

Column	Maximum Value
1	25
2	35
3	20
4	28
5	30

Minimum of {25, 35, 20, 28, 30}= 20

It is observed that the maximum of row minima and the minimum of the column maxima are equal. Hence the given the game has a saddle point. The common value is 20.This indicates that the value V of the game is 20.

Interpretation: The best strategy for player X is strategy 2.
The best strategy for player Y is strategy III.
The game is favorable to player A.

Problem :

Solve the following game:

Player B

Strategies

Player A Strategies		*I*	*II*	*III*	*IV*
	1	1	−6	8	4
	2	3	−7	2	−8
	3	5	−5	−1	0
	4	3	−4	5	7

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-6
2	-8
3	-5
4	-4

Maximum of {-6, -8, -5, -4} = -4

Next consider the maximum of each column

Column	Maximum Value
1	5
2	-4
3	8
4	7

Minimum of {5, -4, 8, 7}= - 4

Since the max {row minima} = min {column maxima}, the game under consideration has a saddle point. The common value is –4. Hence the value of the game is –4.

Interpretation.

The best strategy for player A is strategy 4.

The best strategy for player B is strategy II. Since the value of the game is negative, it is concluded that the game is favorable to player B.

Games with no Saddle point:

2 x 2 zero-sum game When each one of the first player A and the second player B has exactly two strategies, we have a 2 x 2 game.

Motivating point First let us consider an illustrative example**.**

Problem :

Examine whether the following 2 x 2 game has a saddle point

	Player B	
Player A	3	5
	4	2

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	3
2	2

Maximum of {3, 2} = 3

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	5

Minimum of {4, 5}= 4

We see that max {row minima} and min {column maxima} are not equal. Hence thegame has no saddle point

Method of solution of a 2x2 zero-sum game without saddle point**:** Suppose that a 2x2 game has no saddle point. Suppose the game has the following pay-off matrix.

	Player B Strategy	
Player A Strategy	a	b
	c	d

Since this game has no saddle point, the following condition shall hold:

$$Max\{Min\{a,b\},Min\{c,d\}\} \neq Min\{Max\{a,c\},Max\{b,d\}\}$$

In this case, the game is called a mixed game. No strategy of Player A can be called the best strategy for him. Therefore A has to use both of his strategies. Similarly no strategy of Player B can be called the best strategy for him and he has to use both of his strategies.

Let *p* be the probability that Player A will use his first strategy. Then the probability that Player A will use his second strategy is 1-p.If Player B follows his first strategy. Expected value of the pay-off to Player A.

Expected value of the pay-off to Player A

$$= \left\{ \begin{matrix} \text{Expected value of the pay-off to Player A} \\ \text{arising from his first strategy} \end{matrix} \right\} + \left\{ \begin{matrix} \text{Expected value of the pay-off to Player A} \\ \text{arising from his second strategy} \end{matrix} \right\}$$

$$= ap + c(1-p) \longrightarrow (1)$$

In the above equation, note that the expected value is got as the product of the corresponding values

of the pay-off and the probability.

If Player B follows his second strategy

$$\left. \begin{matrix} \text{Expected value of the} \\ \text{pay-off to Player A} \end{matrix} \right\} = bp + d(1-p) \qquad (2)$$

If the expected values in equations (1) and (2) are different, Player B will prefer the minimum of the two expected values that he has to give to player A. Thus B will have a pure strategy.

This contradicts our assumption that the game is a mixed one. Therefore the expected values of the pay-offs to Player A in equations (1) and (2) should be equal. Thus we have the condition

$$
\begin{aligned}
ap + c(1-p) &= bp + d(1-p) \\
ap - bp &= (1-p)[d-c] \\
p(a-b) &= (d-c) - p(d-c) \\
p(a-b) + p(d-c) &= d-c \\
p(a-b+d-c) &= d-c \\
p &= \frac{d-c}{(a+d)-(b+c)} \\
1-p &= \frac{a+d-b-c-d+c}{(a+d)-(b+c)} \\
&= \frac{a-b}{(a+d)-(b+c)}
\end{aligned}
$$

$$
\left\{\begin{matrix}\text{The number of times A} \\ \text{will use first strategy}\end{matrix}\right\} : \left\{\begin{matrix}\text{The number of times A} \\ \text{will use second strategy}\end{matrix}\right\} = \frac{d-c}{(a+d)-(b+c)} : \frac{a-b}{(a+d)-(b+c)}
$$

The expected pay-off to Player A

$$
\begin{aligned}
&= ap + c(1-p) \\
&= c + p(a-c) \\
&= c + \frac{(d-c)(a-c)}{(a+d)-(b+c)} \\
&= \frac{c\{(a+d)-(b+c)\} + (d-c)(a-c)}{(a+d)-(b+c)} \\
&= \frac{ac + cd - bc - c^2 + ad - cd - ac + c^2)}{(a+d)-(b+c)} \\
&= \frac{ad-bc}{(a+d)-(b+c)}
\end{aligned}
$$

Therefore, the value V of the game is

$$
\frac{ad-bc}{(a+d)-(b+c)}
$$

To find the number of times that B will use his first strategy and second strategy:
Let the probability that B will use his first strategy be *r*. Then the probability that B will use his second strategy is *1-r*.

When A use his first strategy

The expected value of loss to Player B with his first strategy = *ar*
The expected value of loss to Player B with his second strategy = *b(1-r)*
Therefore the expected value of loss to B = ar + b(1-r) ⟶ (3)

When A use his second strategy
The expected value of loss to Player B with his first strategy = cr
The expected value of loss to Player B with his second strategy = d(1-r)

Therefore the expected value of loss to B = cr + d(1-r) $\longrightarrow$ (4)
If the two expected values are different then it results in a pure game, which is a contradiction.
Therefore the expected values of loss to Player B in equations (3) and (4) should be equal.
Hence we have the condition

$$ar + b(1-r) = cr + d(1-r)$$
$$ar + b - br = cr + d - dr$$
$$ar - br - cr + dr = d - b$$
$$r(a - b - c + d) = d - b$$
$$r = \frac{d-b}{a-b-c+d}$$
$$= \frac{d-b}{(a+d)-(b+c)}$$

Problem:
Solve the following game

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix} \end{array}$$

Solution:
First consider the row minima

Row	Minimum Value
1	2
2	1

Maximum of {2, 1} = 2
Next consider the maximum of each column

Column	Maximum Value
1	4
2	5

We see that Max {row minima} ≠ min {column maxima}
So the game has no saddle point. Therefore it is a mixed game.
We have a = 2, b = 5, c = 4 and d = 1.
Let *p* be the probability that player X will use his first strategy. We have

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$= \frac{1-4}{(2+1)-(5+4)}$$
$$= \frac{-3}{3-9}$$
$$= \frac{-3}{-6}$$
$$= \frac{1}{2}$$

The probability that player *X* will use his second strategy is $1\text{-}p = 1\text{-}\frac{1}{2} = \frac{1}{2}$.

Value of the game V = $\frac{ad-bc}{(a+d)-(b+c)} = \frac{2-20}{3-9} = \frac{-18}{-6} = 3$.

Let *r* be the probability that Player Y will use his first strategy. Then the probability that Y will use his second strategy is *(1-r)*. We have

$$r = \frac{d-b}{(a+d)-(b+c)}$$
$$= \frac{1-5}{(2+1)-(5+4)}$$
$$= \frac{-4}{3-9}$$
$$= \frac{-4}{-6}$$
$$= \frac{2}{3}$$
$$1-r = 1-\frac{2}{3} = \frac{1}{3}$$

Interpretation.

$$p : (1\text{-}p) = \frac{1}{2} : \frac{1}{2}$$

Therefore, out of 2 trials, player X will use his first strategy once and his second strategy once.

$$r : (1\text{-}r) = \frac{2}{3} : \frac{1}{3}$$

Therefore, out of 3 trials, player Y will use his first strategy twice and his second strategy once.

The Principle of Dominance:

In the previous lesson, we have discussed the method of solution of a game without a saddle point. While solving a game without a saddle point, one comes across the phenomenon of the dominance of a row over another row or a column over another column in the pay-off matrix of the game. Such a situation is discussed in the sequel.In a given pay-off matrix A, we say that the ith row dominates the k^{th} row if

$a_{ij} \geq a_{kj}$ for all j = 1,2,…,n and $a_{ij} > a_{kj}$ for at least one j.

In this case, the player B will loose more by choosing the strategy for the q^{th} column than by choosing the strategy for the p^{th} column. So he will never use the strategy corresponding to the q^{th} column. When dominance of a row (or a column) in the pay-off matrix occurs, we can delete a row (or a column) from that matrix and arrive at a reduced matrix. This principle of dominance can be used in the determination of the solution for a given game.

Let us consider an illustrative example involving the phenomenon of dominance in a game.

Problem :

Solve the game with the following pay-off matrix:

Player B

$$\text{Player A}\quad \begin{matrix} & I & II & III & IV \\ 1 & 4 & 2 & 3 & 6 \\ 2 & 3 & 4 & 7 & 5 \\ 3 & 6 & 3 & 5 & 4 \end{matrix}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	2
2	3
3	3

Maximum of {2, 3, 3} = 3

Next consider the maximum of each column.

Column	Maximum Value
1	6
2	4
3	7
4	6

Minimum of {6, 4, 7, 6}= 4

The following condition holds:
Max {row minima}≠= min {column maxima}
Therefore we see that there is no saddle point for the game under consideration.
Compare columns II and III.

Column II	Column III
2	3
4	7
3	5

We see that each element in column III is greater than the corresponding element in column II. The choice is for player B. Since column II dominates column III, player B will discard his strategy 3.Now we have the reduced game

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{matrix} I & II & IV \end{matrix} \\ \begin{bmatrix} 4 & 2 & 6 \\ 3 & 4 & 5 \\ 6 & 3 & 4 \end{bmatrix} \end{array}$$

For this matrix again, there is no saddle point. Column II dominates column IV. The choice is for player B. So player B will give up his strategy 4
The game reduces to the following:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{matrix} I & II \end{matrix} \\ \begin{bmatrix} 4 & 2 \\ 3 & 4 \\ 6 & 3 \end{bmatrix} \end{array}$$

This matrix has no saddle point.
The third row dominates the first row. The choice is for player A. He will give up his strategy 1 and retain strategy 3. The game reduces to the following

$$\begin{bmatrix} 3 & 4 \\ 6 & 3 \end{bmatrix}$$

Again, there is no saddle point. We have a 2x2 matrix. Take this matrix as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then we have a = 3, b = 4, c = 6 and d = 3. Use the formulae for p, 1-p, r, 1-r and V.

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$= \frac{3-6}{(3+3)-(6+4)}$$
$$= \frac{-3}{6-10}$$
$$= \frac{-3}{-4}$$
$$= \frac{3}{4}$$

$$1-p = 1-\frac{3}{4} = \frac{1}{4}$$

$$r = \frac{d-b}{(a+d)-(b+c)}$$
$$= \frac{3-4}{(3+3)-(6+4)}$$
$$= \frac{-1}{6-10}$$
$$= \frac{-1}{-4}$$
$$= \frac{1}{4}$$

$$1-r = 1-\frac{1}{4} = \frac{3}{4}$$

The value of the game

$$V = \frac{ad-bc}{(a+d)-(b+c)}$$
$$= \frac{3x3-4x6}{-4}$$
$$= \frac{-15}{-4}$$
$$= \frac{15}{4}$$

Thus, $X = \left(\frac{3}{4}, \frac{1}{4}, 0, 0\right)$ and $Y = \left(\frac{1}{4}, \frac{3}{4}, 0, 0\right)$ are the optimal strategies.

Method of convex linear combination :

A strategy, say s, can also be dominated if it is inferior to a convex linear combination of several other pure strategies. In this case if the domination is strict, then the strategy s can be deleted. If strategy s dominates the convex linear combination of some other pure strategies, then one of the pure strategies involved in the combination may be deleted. The domination will be decided as per the above rules. Let us consider an example to illustrate this case.

Problem:

Solve the game with the following pay-off matrix for firm A:

$$\text{Firm A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \overset{\text{Firm B}}{\begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \\ \hline 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \\ 4 & -3 & 5 & 6 & 3 \\ 4 & -1 & 5 & 7 & 3 \end{array}}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-2
2	0
3	-6
4	-3
5	-1

Maximum of {-2, 0, -6, -3, -1} = 0

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	8
3	6
4	8
5	6

Minimum of { 4, 8, 6, 8, 6}= 4

Hence,

Maximum of {row minima}$\neq$ minimum of {column maxima}.

So we see that there is no saddle point. Compare the second row with the fifth row. Each element in the second row exceeds the corresponding element in the fifth row. Therefore, A_2 dominates A_5 . The choice is for firm A. It will retain strategyA_2 and give up strategy A_5 .Therefore the game reduces to the following.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \\ 4 & -3 & 5 & 6 & 3 \end{bmatrix} \end{array}$$

Compare the second and fourth rows. We see that A_2 dominates A_4 . So, firm A will retain the strategy A_2 and give up the strategy A_4 . Thus the game reduces to the following:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \end{bmatrix} \end{array}$$

Compare the first and fifth columns. It is observed that B1 dominates B5. The choice is for firm B. It will retain the strategy B_1 and give up the strategy B_5 . Thus the game reduces to the Following

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 \\ 4 & 0 & 6 & 8 \\ -2 & -6 & -4 & 4 \end{bmatrix} \end{array}$$

Compare the first and fourth columns. We notice that B1 dominates B4. So firm B will discard the strategy B_4 and retain the strategy B_1 . Thus the game reduces to the following

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 \\ 4 & 0 & 6 \\ -2 & -6 & -4 \end{bmatrix} \end{array}$$

For this reduced game, we check that there is no saddle point. Now none of the pure strategies of firms A and B is inferior to any of their other strategies. But, we observe that convex linear combination of the strategies B_2 and B_3dominates B_1 , i.e. the averages of payoffs due to strategies B_2 and B_3,

$$\left\{\frac{8-2}{2},\frac{0+6}{2},\frac{-6-4}{2}\right\}=\{3,3,-5\}$$

dominate B_1 . Thus B_1 may be omitted from consideration. So we have the reduced matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cc} B_2 & B_3 \end{array} \\ \begin{bmatrix} 8 & -2 \\ 0 & 6 \\ -6 & -4 \end{bmatrix} \end{array}$$

Here, the average of the pay-offs due to strategies A_1 and A_2 of firm A, i.e.

$\left\{\frac{8+0}{2}, \frac{-2+6}{2}\right\} = \{4, 2\}$ dominates the pay-off due to A_3. So we get a new reduced 2x2 pay-off matrix

Firm B's strategy

$$\text{Firm A's strategy} \quad \begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{cc} B_2 & B_3 \end{array} \\ \begin{bmatrix} 8 & -2 \\ 0 & 6 \end{bmatrix} \end{array}$$

We have a = 8, b = -2, c = 0 and d = 6.

$$p = \frac{d-c}{(a+d)-(b+c)}$$

$$= \frac{6-0}{(6+8)-(-2+0)}$$

$$= \frac{6}{16}$$

$$= \frac{3}{8}$$

$$1-p = 1-\frac{3}{8} = \frac{5}{8}$$

$$r = \frac{d-b}{(a+d)-(b+c)}$$

$$= \frac{6-(-2)}{16}$$

$$= \frac{8}{16}$$

$$= \frac{1}{2}$$

$$1-r = 1-\frac{1}{2} = \frac{1}{2}$$

. Value of the game

$$V = \frac{ad - bc}{(a+d)-(b+c)}$$

$$= \frac{6x8 - 0x(-2)}{16}$$

$$= \frac{48}{16} = 3$$

So the optimal strategies are

$$A = \left\{\frac{3}{8}, \frac{5}{8}, 0, 0, 0\right\} \text{ and } B = \left\{0, \frac{1}{2}, \frac{1}{2}, 0, 0\right\}.$$

The value of the game = 3. Thus the game is favourable to firm A.

Problem:

For the game with the following pay-off matrix, determine the saddle point

Player B

$$\text{Player A} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{bmatrix} 2 & -1 & 0 & -3 \\ 1 & 0 & 3 & 2 \\ -3 & -2 & -1 & 4 \end{bmatrix} \end{array}$$

Solution:

$$\begin{array}{ccc} \textit{Column II} & \textit{Column III} & \\ \begin{array}{c|c|} 1 & -1 \\ 2 & 0 \\ 3 & -2 \end{array} & \begin{array}{|c|} 0 \\ 3 \\ -1 \end{array} & \begin{array}{c} 0 > -1 \\ 3 > \ \ 0 \\ -1 > -2 \end{array} \end{array}$$

The choice is with the player B. He has to choose between strategies II and III. He will lose more in strategy III than in strategy II, irrespective of what strategy is followed by A. So he will drop strategy III and retain strategy II. Now the given game reduces to the following game.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} I & II & IV \end{array} \\ \begin{bmatrix} 2 & -1 & -3 \\ 1 & 0 & 2 \\ -3 & -2 & 4 \end{bmatrix} \end{array}$$

Consider the rows and columns of this matrix

Row minimum:

I Row	:	-3	
II Row	:	0	Maximum of {-3, 0, -3} = 0
III Row	:	-3	

Column maximum:

I Column	:	2	
II Column	:	0	Minimum of {2, 0, 4} = 0
III Column	:	4	

Interpretation: No player gains and no player loses. i.e., The game is not favourable to any player. i.e. It is a fair game.

Problem:

Solve the game

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 4 & 8 & 6 \\ 6 & 2 & 10 \\ 4 & 5 & 7 \end{bmatrix}}$$

Solution:

First consider the minimum of each row

Row	Minimum
1	4
2	2
3	4

Maximum of {4, 2, 4} = 4

Next, consider the maximum of each column.

Column	Maximum
1	6
2	8
3	10

Minimum of {6, 8, 10} = 6

Since Maximum of { Row Minima} and Minimum of { Column Maxima } are different, it follows that the given game has no saddle point.

Denote the strategies of player A by A_1, A_2 , A_3 . Denote the strategies of player B by B_1 ,B_2 ,B_3
.

Compare the first and third columns of the given matrix.

$$\begin{matrix} B_1 & B_3 \\ \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} & \begin{bmatrix} 6 \\ 10 \\ 7 \end{bmatrix} \end{matrix}$$

The pay-offs in B_3 are greater than or equal to the corresponding pay-offs in B_1 . The player B has to make a choice between his strategies 1 and 3. He will lose more if he follows strategy 3 rather than strategy 1. Therefore he will give up strategy 3 and retain strategy 1. Consequently, the given game is transformed into the following game:

$$\begin{matrix} & B_1 & B_2 \\ A_1 & 4 & 8 \\ A_2 & 6 & 2 \\ A_3 & 4 & 5 \end{matrix}$$

Compare the first and third rows of the above matrix.

$$\begin{matrix} & B_1 & B_2 \\ A_1 & 4 & 8 \\ A_3 & 4 & 5 \end{matrix}$$

The pay-offs in A_1 are greater than or equal to the corresponding pay-offs in A_2 . The player A has to make a choice between his strategies 1 and 3. He will gain more if he follows strategy 1 rather than strategy 3. Therefore he will retain strategy 1 and give up strategy 3.Now the given game is transformed into the following game.

$$\begin{matrix} & B_1 & B_2 \\ A_1 & 4 & 8 \\ A_2 & 6 & 2 \end{matrix}$$

It is a 2x2 game. Consider the row minima

Row	Minimum
1	4
2	2

Maximum of {4, 2} = 4

Next, consider the maximum of each column

Column	Maximum
1	6
2	8

Minimum of {6, 8} = 6

Maximum {row minima} and Minimum {column maxima } are not equal.Therefore, the reduced game has no saddle point. So, it is a mixed game

Take $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix}$. We have a = 4, b = 8, c = 6 and d = 2.

The probability that player A will use his first strategy is p. This is calculated as

$$p = \frac{d-c}{(a+d)-(b+c)}$$

$$= \frac{2-6}{(4+2)-(8+6)}$$

$$= \frac{-4}{6-14}$$

$$= \frac{-4}{-8} = \frac{1}{2}$$

The probability that player B will use his first strategy is r. This is calculated as

$$r = \frac{d-b}{(a+d)-(b+c)}$$

$$= \frac{2-8}{-8}$$

$$= \frac{-6}{-8}$$

$$= \frac{3}{4}$$

Value of the game is V. This is calculated as

$$V = \frac{ad-bc}{(a+d)-(b+c)}$$

$$= \frac{4x2-8x6}{-8}$$

$$= \frac{8-48}{-8}$$

$$= \frac{-40}{-8} = 5$$

Interpretation
Out of 3 trials, player A will use strategy 1 once and strategy 2 once. Out of 4 trials, player B will use strategy 1 thrice and strategy 2 once. The game is favorable to player A.

Problem:
Solve the game with the following pay-off matrix.**(Dividing a game into sub-games)**

Player B

$$\text{Player A}\quad \begin{array}{c} \\ I \\ II \\ III \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -4 & 6 & 3 \\ -3 & 3 & 4 \\ 2 & -3 & 4 \end{bmatrix} \end{array}$$

Solution:
First, consider the row minima.

Row	Minimum
1	-4
2	-3
3	-3

Maximum of {-4, -3, -3} = -3

Next, consider the column maxima.

Column	Maximum
1	2
2	6
3	4

Minimum of {2, 6, 4} = 2

We see that Maximum of { row minima} ≠ Minimum of { column maxima}.
So the game has no saddle point. Hence it is a mixed game. Compare the first and third Columns.

$$\begin{array}{ccc} I\ Column & III\ Column & \\ \begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} & \begin{array}{c} -4 \le 3 \\ -3 \le 4 \\ 2 \le 4 \end{array} \end{array}$$

We assert that Player B will retain the first strategy and give up the third strategy. We get the following reduced matrix

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \\ 2 & -3 \end{bmatrix}$$

We check that it is a game with no saddle point.

Sub games : Let us consider the 2x2 sub games. They are:

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$$

First, take the sub game

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}$$

Compare the first and second columns. We see that $-4 \le 6$ and $-3 \le 3$. Therefore, the game reduces to $\begin{bmatrix} -4 \\ -3 \end{bmatrix}$. Since $-4 < -3$, it further reduces to -3.

Next, consider the sub game

$$\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}$$

We see that it is a game with no saddle point. Take a = -4, b = 6, c = 2, d = -3. Then the value of the game is

$$V = \frac{ad - bc}{(a+d)-(b+c)}$$

$$= \frac{(-4)(-3)-(6)(2)}{(-4+3)-(6+2)}$$

$$= 0$$

Next, take the sub game $\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$. In this case we have a = -3, b = 3, c = 2 and d = -3. The value of the game is obtained as

$$V = \frac{ad - bc}{(a+d)-(b+c)}$$

$$= \frac{(-3)(-3)-(3)(2)}{(-3-3)-(3+2)}$$

$$= \frac{9-6}{-6-5} = -\frac{3}{11}$$

Let us tabulate the results as follows:

Sub game	Value
$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}$	-3
$\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}$	0
$\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$	$-\frac{3}{11}$

The value of 0 will be preferred by the player A. For this value, the first and third strategies of A correspond while the first and second strategies of the player B correspond to the value 0 of the game. So it is a fair game.

<u>Graphical solution of a 2x2 game with no saddle point:</u>

Problem:
Consider the game with the following pay-off matrix.

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}}$$

Solution: First consider the row minima.

Row	Minimum
1	2
2	1

Maximum of {2, 1} = 2.

Next, consider the column maxima.

Column	Maximum
1	4
2	5

Minimum of {4, 5} = 4.

We see that Maximum { row minima} ≠ Minimum { column maxima }
So, the game has no saddle point. It is a mixed game.
Equations involving probability and expected value:
Let p be the probability that player A will use his first strategy.
Then the probability that A will use his second strategy is **1--p**.
Let E be the expected value of pay-off to player A.
When B uses his first strategy
The expected value of pay-off to player A is given by

$$E = 2p + 4(1-p)$$
$$= 2p + 4 - 4p$$
$$= 4 - 2p \quad \longrightarrow \quad (1)$$

When B uses his second strategy
The expected value of pay-off to player A is given by

$$E = 5p + 1(1-p)$$
$$= 5p + 1 - p$$
$$= 4p + 1 \quad \longrightarrow \quad (2)$$

Consider equations (1) and (2). For plotting the two equations on a graph sheet, get some points on them as follows: E = -2p+4

p	0	1	0.5
E	4	2	3

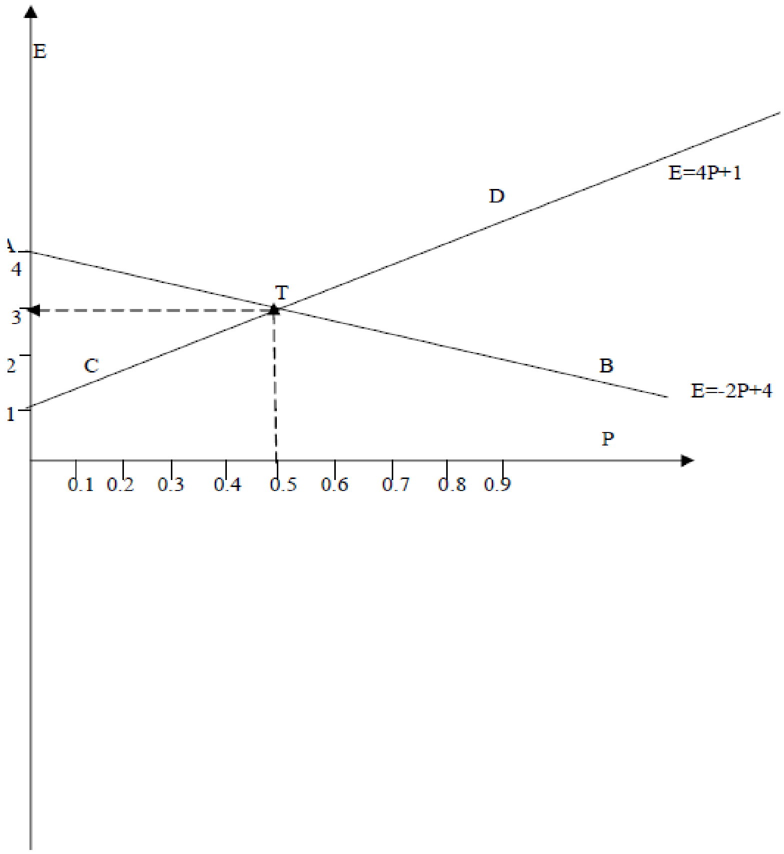
E = 4p+1

p	0	1	0.5
E	1	5	3

Graphical solution:

Procedure: Take probability and expected value along two rectangular axes in a graph sheet. Draw two straight lines given by the two equations (1) and (2). Determine the point of intersection of the two straight lines in the graph. This will give the common solution of the two equations (1) and (2). Thus we would obtain the value of the game.

Represent the two equations by the two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have p = 0.5 and E = 3. Therefore, the value V of the game is 3.



Problem:

Solve the following game by graphical method.

$$\text{Player A}\ \overset{\text{Player B}}{\begin{bmatrix} -18 & 2 \\ 6 & -4 \end{bmatrix}}$$

Solution:

First consider the row minima.

Row	Minimum
1	- 18
2	- 4

Maximum of {-18, - 4} = - 4.

Next, consider the column maxima.

Column	Maximum
1	6
2	2

Minimum of {6, 2} = 2.

We see that Maximum {row minima} ≠ Minimum { column maxima }So, the game has no saddle point. It is a mixed game. Let p be the probability that player A will use his first strategy. Then the probability that A will use his second strategy is 1- p.

When B uses his first strategy. The expected value of pay-off to player A is given by

$$E = -18p + 6(1-p)$$
$$= -18p + 6 - 6p$$
$$= -24p + 6$$

E = -24 p + 6

p	0	1	0.5
E	6	-18	-6

$$E = 2p - 4(1-p)$$
$$= 2p - 4 + 4p$$
$$= 6p - 4$$

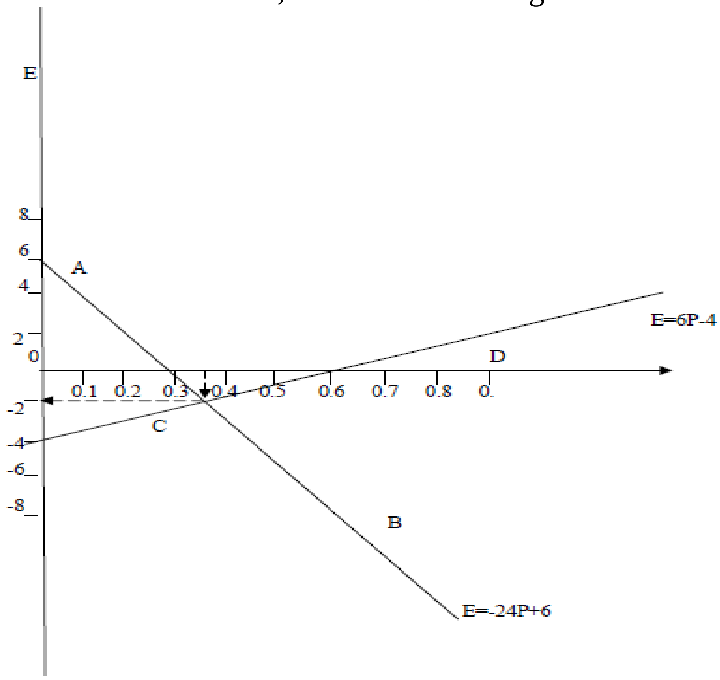
E = 6p-4

p	0	1	0.5
E	-4	2	-1

Graphical solution:

Take probability and expected value along two rectangular axes in a graph sheet. Draw two straight lines given by the two equations (1) and (2). Determine the point of intersection of the two straight lines in the graph. This will provide the common solution of the two equations (1) and (2). Thus we would get the value of the game.

Represent the two equations by the two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have p = 1/ 3 and E = -2. Therefore, the value V of the game is -2.



Tutorial Questions

1. a)Explain the terms i)Rectangular games ii) type of Strategies

   b) Solve the following game graphically where pay off matrix for player A has been prepared

1	5	-7	4	2
2	4	9	-3	1

2. a)Explain the terms
   i) Maxmin criteria and Minimax criteria ii) Strategies: Pure and mixed strategies.

   b) Solve the following game graphically

	Player B		
Player A	B_1	B_2	B_3
A_1	1	3	11
A_2	8	5	2

3. a)What are characteristics of a game?

   b Reduce the following Game by dominance and the fit the game value

PlayerA					
		I	II	III	IV
	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

4. a) Obtain the optimal strategies for both pensions and the value of the game for two persons zero sum game whose payoff matrix is as follows.

		player-B	
		B1	B2
Player-A	A1	1	-3
	A2	3	5
	A3	-1	6
	A4	4	1
	A5	2	2
	A6	-5	0

   b) Explain pay of matrix and types of strategy in game theory?

Assignment Questions

1 a)What are characteristics of a game?

b) Reduce the following Game by dominance and the fid the game value

PlayerA					
		I	II	III	IV
	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

2. a)Explain the terms i)Rectangular games ii) type of Strategies

b) Solve the following game graphically where pay off matrix for player A has been prepared

1	5	-7	4	2
2	4	9	-3	1

UNIT 4

Replacement Analysis

&

Inventory

UNIT- 4
Replacement Problems

Introduction:

The replacement problems are concerned with the situations that arise when some items such as men, machines and usable things etc need replacement due to their decreased efficiency, failure or breakdown. Such decreased efficiency or complete breakdown may either be gradual or all of a sudden.

If a firm wants to survive the competition it has to decide on whether to replace the out dated equipment or to retain it, by taking the cost of maintenance and operation into account. There are two basic reasons for considering the replacement of an equipment.

They are (i) Physical impairment or malfunctioning of various parts.

(ii) Obsolescence of the equipment.

The physical impairment refers only to changes in the physical condition of the equipment itself. This will lead to decline in the value of service rendered by the equipment, increased operating cost of the equipments, increased maintenance cost of the equipment or the combination of these costs. Obsolescence is caused due to improvement in the existing Tools and machinery mainly when the technology becomes advanced therefore, it becomes uneconomical to continue production with the same equipment under any of the above situations. Hence the equipments are to be periodically replaced.

Sometimes, the capacity of existing facilities may be in adequate to meet the current demand. Under such cases, the following two alternatives will be considered.

1. Replacement of the existing equipment with a new one
2. Argument the existing one with an additional equipments.

Type of Maintenance:

Maintenance activity can be classified into two types

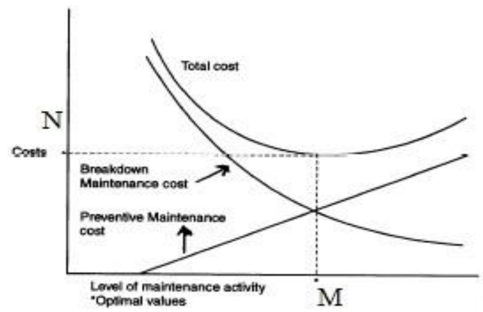
i) Preventive Maintenance

ii) Breakdown Maintenance

Preventive maintenance (PN) is the periodical inspection and service which are aimed to detect potential failures and perform minor adjustments a requires which will prevent major operating problem in future. Breakdown maintenance is the repair which is generally done after the equipment breaks down. It is offer an emergency which will have an associated penalty in terms of increasing the cost of maintenance and downtime cost of equipment, Preventive maintenance will reduce such costs up-to a certain extent . Beyond that the cost of preventive maintenance will be more when compared to the cost of the breakdown maintenance.

Total cost = Preventive maintenance cost + Breakdown maintenance cost.

This total cost will go on decreasing up-to P with an increase in the level of maintenance up-to apoint, beyond which the total cost will start increasing from P. The level of maintenance corresponding to the minimum total cost at P is the Optional level of maintenance this concept is illustrated in the follows diagram



The points M and N denote optimal level of maintenance and optimal cost respectively
Types of replacement problem : The replacement problem can be classified into two categories.
i) Replacement of assets that deteriorate with time (replacement due to gradual failure, due to wear and tear of the components of the machines) This can be further classified into the following types.
a) Determination of economic type of an asset.
b) Replacement of an existing asset with a new asset.
ii) Simple probabilistic model for assets which will fail completely (replacement due to sudden failure).

Determination of Economic Life of an asset

Any asset will have the following cost components
i) Capital recovery cost (average first cost), Computed form the first cost (Purchase price) of the asset.
ii) Average operating and maintenance cost.
iii) Total cost which is the sum of capital recovery cost (average first cost) and average operating and maintenance cost.
A typical shape of each of the above cost with respect to life of the asset is shown below.

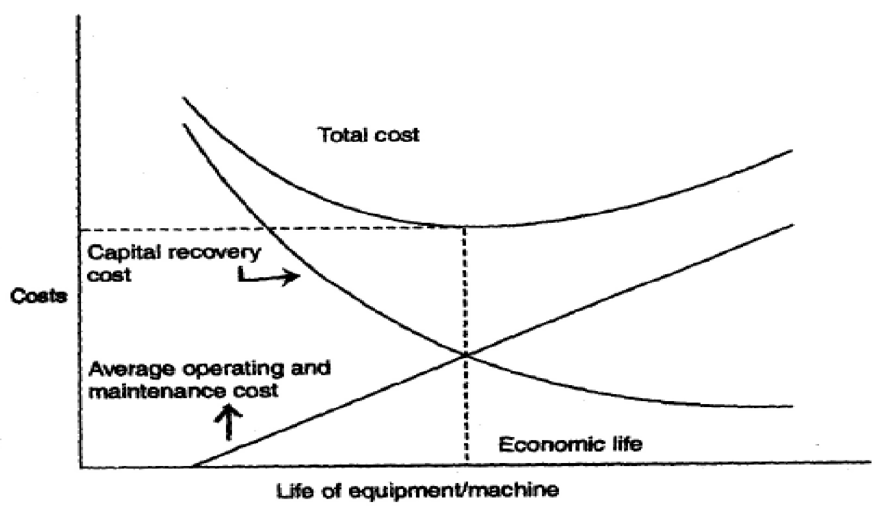


Chart showing economic life.

From figure, when the life of the machine increases, it is clear that the capital recovery cost (average first cost) goes on decreasing and the average operating and maintenance cost goes on increasing. From the beginning the total cost goes on decreasing upto a particular life of the asset and then it starts increasing. The point P were the total cost in the minimum is called the Economic life of the asset. To solve problems under replacement, we consider the basics of interest formula.

Time value of money does not change

If the value of money does not change with time, then the user of the equipment does not need to pay interest on his investments. We wish to determine the optimal time to replace the equipment. If the value of money does not change with time, then the user of the equipment does not need to pay interest on his investments. We wish to determine the optimal time to replace the equipment.

We make use of the following notations:

C = Capital cost of the equipment

S = Scrap value of the equipment

n = Number of years that the equipment would be in use

Cm = Maintenance cost function.

ATC = Average total annual costent.We make use of the following notations:

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n = Number of years that the equipment would be in use

Cm = Maintenance cost function.

ATC = Average total annual cost

Two possibilities are there

(i) Time *t* is a continuous random variable In this case the deterioration of the equipment is being monitored continuously. The

total cost of the equipment during n years of use is given by

TC = Capital cost - Scrap value + Maintenance cost

$$= C - S + \int_0^n C_m(t)\, dt$$

$$\therefore A(n) = \frac{1}{n} TC = \frac{C-S}{n} + \frac{1}{n}\int_0^n C_m(t)\, dt$$

$$\text{For minimum cost, } \frac{d}{dn}A(n) = 0$$

$$\therefore -\frac{C-S}{n^2} - \frac{1}{n^2}\int_0^n C_m(t)dt + \frac{1}{n}C_m(n) = 0$$

$$\therefore C_m = \frac{C-S}{n^2} + \frac{1}{n^2}\int_0^n C_m(n)\, dt = A(n)$$

$$\text{And } \frac{d^2A(n)}{dn^2} \geq 0 \text{ at } C_m(n) = A(n)$$

i.e., when the maintenance cost becomes equal to the average annual cost, the decision should be to replace the equipment.

(ii) Time *t* is a discrete random variable

In this case

$$A(n) = \frac{1}{n}TC = \frac{C - S}{n} + \frac{1}{n}\sum_{0}^{n} C_m$$

A (*n*) is Minimum when

$$A(n+1) \geq A(n) \text{ and } A(n-1) \geq A(n)$$

$$\text{Or, } A(n+1) - A(n) \geq 0 \textit{ and } A(n) - A(n-1) \leq 0$$

$$A(n+1) - A(n)\frac{1}{n+1}\left(C - S + \sum_{0}^{n} C_m(t)\right) + \frac{1}{n+1}C_m(n+1) - A(n)$$

$$\frac{n}{n+1}A(n) + \frac{1}{n+1}C_m(n+1) - A(n) \geq 0$$

$$\therefore A(n+1) \geq A(n)$$

Similarly

$$A(n) - A(n-1) \leq 0$$

$$\therefore C_m(n) \leq C_m(n-1)$$

Thus the optimal policy is Replace the equipment at the end of n years if the maintenance cost in the (n+1) th year is more than the average total cost in the nth year and the nth year's maintenance cost is less than previous year's average total cost.

Present worth factor denoted by (P/F, i,n). If an amount P is invested now with amount earning interest at the rate i per year, then the future sum (F) accumulated after n years can be obtained.

P - Principal sum at year Zero

F - Future sum of P at the end of the nth year

i - Annual interest rate

n - Number of interest periods.

Then the formula for future sum $F = P(1 + i)^n$

$P = F/(1 + i)^n = F x$ (present worth factor)

If A is the annual equivalent amount which occurs at the end of every year from year one through n years is given by

$$A = \frac{P \times i(1+i)^n}{(1+i)^n - 1}$$

$$= P(A/P, i, n)$$

= P x equal payment series capital recovery factor

Problem;

The cost of equipment is Rs. 62,000 and its scrap value is Rs. 2,000. The life of the equipment is 8 years. The maintenance costs for each year are as given below:

Year	1	2	3	4	5	6	7	8
Maintenance Cost in Rs.	1000	2000	3500	5000	8000	11000	16000	24000

When the equipment should be replaced?

Solution:-

C = 62,000/-

As the avg. yearly cost is minimum for 6th year the equipment should be replace after 6 year.

Year **n**	Resale Price **S**	Maintenance Cost C_m	Cumulative Maintenance Cost $\sum C_m$	Total Cost $TC = C - S + \sum C_m$	Annual Total Cost $ATC = \frac{TC}{n}$
1	2000	1000	1000	61000	61000
2	2000	2000	3000	63000	31500
3	2000	3500	6500	65000	21666.6
4	2000	5000	11500	71500	17875
5	2000	8000	19500	79500	15900
6	**2000**	**11000**	**30500**	**90500**	**15083.3**
7	2000	16000	46500	106500	15214.2
8	24000				

Problem:

(a) Machine A cost Rs. 36,000. Annual operating costs are Rs. 800 for the first year, and then increase by Rs. 8000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy if followed, what will be the yearly cost of owning and operating the machine?
(b) Machine B costs Rs. 40,000. Annual operating costs Rs. 1,600 for the first year, and then increase by Rs. 3,200 every year. You now have a machine of type A which is one year old. Should you replace it with B, if so when? Assume that both machines have no resale value

Solution:-

(a) Machine A

C = 36,000/- □As the avg. yearly cost is minimum for 3rd year for machine A, machine A should be replace after 3 year.

Avg. yearly cost for operating & owing the machine A is Rs. 20,800.

□The avg. cost per year of operating & owing the machine B is less that of machine A.

Year **n**	Resale Price **S**	Maintenance Cost **C_m**	Cumulative Maintenance Cost **$\sum C_m$**	Total Cost **$TC = C - S + \sum C_m$**	Annual Total Cost **$ATC = \frac{TC}{n}$**
1	0	800	800	36800	36800
2	0	8800	9600	45600	22800
3	**0**	**16800**	**26400**	**62400**	**20800**
4	0	24800	51200	87200	21800

(b) Machine B

C = 40,000/- Machine A should be replaced with machine B. As the cost of using machine A in 3rd year is more than avg. yearly cost of operating & owing the machine.

□ Machine A should be replaced machine B after 2 years. i.e. 1 year from now because of machine A is already 1 year old.

Year n	Resale Price S	Maintenance Cost C_m	Cumulative Maintenance Cost $\sum C_m$	Total Cost $TC=C-S+\sum C_m$	Annual Total Cost $ATC = \frac{TC}{n}$
1	0	1600	1600	41600	41600
2	0	4800	6400	46400	23200
3	0	8000	14400	54400	18133.3
4	0	11230	25600	65600	16400
5	**0**	**14400**	**40000**	**80000**	**16000**
6	0	17600	57600	97600	16266.6

N^{th} year	Cost of N^{th} year (Rs.)
2	45600-36800=8800
3	62400-45600=16800

Problem :

A firm pays Rs. 10,000 for its equipment. Their operating and maintenance costs are about Rs. 2500 per year for the first two years and then go up by approximately Rs. 1,500 per year. When such equipment replaced? The discount rate is 10% per year.

Solution:-

$$d = \frac{1}{1+i} = \frac{1}{1+0.1} = 0.909$$

C = 10,00 0/- i = 0.10

Year n	C_m	Discount Factor d^{n-1}	Discounted Maintenance Cost $C_m * d^{n-1}$	Discounted Cumulative Maintenance Cost $\sum C_m * d^{n-1}$	$TC=C-S+\sum C_m * d^{n-1}$	$\sum d^{n-1}$	$ATC = \frac{TC}{\sum d^{n-1}}$
1	2500	1	2500	2500	12500	1	12500
2	2500	0.909	2272.5	4772.5	14772.5	1.909	7738.3
3	4000	0.826	3304	8076.5	18076.5	2.735	6609.3
4	**5500**	**0.751**	**4130.5**	**12207**	**22207**	**3.486**	**6370.3**
5	7000	0.683	4781	16988	26988	4.169	6473.4

As the avg. yearly cost is minimum for 4th year the equipment should be replace after 4 year

Problem :

The following mortality rates have been observation for certain type of light bulbs There are 1000 bulbs in use and it costs Rs 10 to replace an individual bulb which has burnt out. If all bulbs were replaced simultaneously, it would cost Rs 2.5 per bulbs. It is proposed to replace all the bulbs at fixed interval, and individually those which fail between the intervals. What would be the best policy to adopt?

Month	1	2	3	4	5
Percent failing by month end	10	25	50	80	100

Solution:

Month i	**Cumulative % failure up to the end of month**	**% failure during the month**	**Probability** P_i **that a new bulb shall fail during the month**
1	10	10	0.10
2	25	15	0.15
3	50	25	0.25
4	80	30	0.30
5	100	20	0.20

Month i	Bulbs failing during i^{th} month	Bulbs replaced until i^{th} month	Cost of Individual Replacement TCI	Cost of Group Replacement TCG	Total Cost TC=TCI+TCG	Average Cost per month ATC = $\frac{TC}{n}$
1	100	100	1000	2500	3500	3500
2	**160**	**260**	**2600**	**2500**	**5100**	**2550**
3	281	541	5410	2500	7910	2636.6
4	377	918	9180	2500	11680	2920
5	349	1267	12670	2500	15170	3034

$$N_0 = 1000$$

$$N_1 = N_0 \times P_1$$

$$= \frac{10}{100} \times 1000$$

$$= 100$$

$$N_2 = N_0 \times P_2 + N_1 \times P_1$$
$$= \frac{15}{100} \times 1000 + \frac{10}{100} \times 100$$
$$= 160$$
$$N_3 = N_0 \times P_3 + N_1 \times P_2 + N_2 \times P_3$$
$$= \frac{25}{100} \times 1000 + \frac{15}{100} \times 100 + \frac{10}{100} \times 160$$
$$= 281$$
$$N_4 = N_0 \times P_4 + N_1 \times P_3 + N_2 \times P_2 + N_3 \times P_1$$
$$= \frac{30}{100} \times 1000 + \frac{25}{100} \times 100 + \frac{15}{100} \times 160 + \frac{10}{100} \times 281$$
$$= 377$$
$$N_5 = N_0 \times P_5 + N_1 \times P_4 + N_2 \times P_3 + N_3 \times P_2 + N_4 \times P_1$$
$$= \frac{20}{100} \times 1000 + \frac{30}{100} \times 100 + \frac{25}{100} \times 160 + \frac{15}{100} \times 281 + \frac{10}{100} \times 377$$
$$= 349$$

$$\text{Avg life} = \sum i \times P_i$$
$$= 1(P_1) + 2(P_2) + 3(P_3) + 4(P_4) + 5(P_5)$$
$$= 1(0.1) + 2(0.15) + 3(0.25) + 4(0.3) + 5(0.2)$$
$$= 3.35 \text{ months}$$

$$\text{No. of bulbs replaced per months} = \frac{1000}{3.35}$$
$$= 298 \text{ bulbs}$$
$$\text{Cost of individual replacement} = 298 \times 10$$
$$= 2980 \text{ Rs.}$$

As cost of group replacement after every 2nd month is less than cost of individual replacement. Group replacement policy after every 2 months is better.

Problem:

A firm is considering replacement of an equipment whose first cost is Rs. 1750 and the scrap value is negligible at any year. Based on experience, it is found that maintenance cost is zero during the first year and it increases by Rs. 100 every year thereafter.

(i) When should be the equipment replaced if

a) i = 0%

b) i = 12%

Solution :

Given the first cost = Rs 1750 and the maintenance cost is Rs. Zero during the first years and then increases by Rs. 100 every year thereafter. Then the following table shows the calculation.

(a.) Calculations to determine Economic life (a) First cost Rs. 1750. Interest rate = 0%

End of year (n)	Maintenance cost at end of year	Summation of maintenance Cost	Average cost of maintenance through the given year	Average first cost if replaced at the given year and	Average total cost through the given year
A	B (Rs)	C (Rs)	D (in Rs)	E (Rs)	F (Rs)
		C = ΣB	C/A	$\frac{1750}{A}$	D + E
1	0	0	0	1750	1750
2	100	100	50	875	925
3	200	300	100	583	683
4	300	600	150	438	588
5	400	1000	200	350	550
6	**500**	**1500**	**250**	**292**	**542**
7	600	2100	300	250	550
8	700	2800	350	219	569

The value corresponding to any end-of-year (n) in Column F represents the average total cost of using the equipment till the end of that particulars year.

In this problem, the average total cost decreases till the end of the year 6 and then it increases. Hence the optimal replacement period is 6 years ie the economic life of the equipment is 6 years .

When interest rate i = 12% When the interest rate is more than 0% the steps to get the economic life are summarized in the following table.

(b.)Calculation to determine Economic life First Cost = Rs. 1750 Interest rate = 12%

End of year (n)	Maintenance cost at end of years	(P/F,12v,n)	Present worth as beginning of years 1 of maintenance costs	Summation of present worth of maintenance costs through the given year	Present simulator maintenance cost and first cost	(A/P, 12%,n) $=\frac{i(1+i)^n}{(1+i)^n-1}$ G	Annual equipment total cost through the giver year
A	B	C	D	E	F	G	H
	B (iR)	C = $\frac{1}{(1+12/100)^n}$	BxC	ΣD	E+ Rs. 1750		FxG
1	0	0.8929	0	0	1750	1.1200	1960
2	100	0.7972	79.72	79.72	1829.72	0.5917	1082.6
3	200	0.7118	142.36	222.08	1972.08	0.4163	820.9
4	300	0.6355	190.65	412.73	2162.73	0.3292	711.9
5	400	0.5674	226.96	639.69	2389.69	0.2774	662.9
6	500	0.5066	253.30	892.99	2642.99	0.2432	642.7
7	600	0.4524	271.44	1164.43	2914.430	0.2191	638.5
8	700	0.4039	282.73	1447.16	3197.16	0.2013	680.7

Identify the end of year for which the annual equivalent total cost is minimum in column. In this problem the annual equivalent total cost is minimum at the end of year hence the economics life of the equipment is 7 years.

Simple probabilistic model for items which completely fail

Electronic items like bulbs, resistors, tube lights etc. generally fail all of a sudden, instead of gradual failure. The sudden failure of the item results in complete breakdown of the system. The system may contain a collection of such items or just an item like a single tube-light. Hence we use some replacement policy for such items which would minimize the possibility of complete breakdown. The following are the replacement policies which are applicable in these cases.
i) Individual replacement policy : Under this policy, each item is replaced immediately after failure.
ii) Group replacement policy : Under group replacement policy, a decision is made with regard the replacement at what equal internals, all the item are to be replaced simultaneously with a provision to replace the items individually which fail during the fixed group replacement period. Among the two types of replacement polices, we have to decide which replacement policy we have to follow. Whether individual replacement policy is better than group replacement policy. with regard to economic point of view. To decide this, each of the replacement policy is calculated and the most economic one is selected for implementation.

Inventory

Introduction :

Simply inventory is a stock of physical assets. The physical assets have some economic value, which can be either in the form of material, men or money. Inventory is also called as an idle resource as long as it is not utilized. Inventory may be regarded as those goods which are procured, stored and used for day to day functioning of the organization.

Inventory can be in the form of physical resource such as raw materials, semi-finished goods used in the process of production, finished goods which are ready for delivery to the consumers, human resources, or financial resources such as working capital etc.

Thus, inventory control is the technique of maintaining stock items at desired levels. In other words, inventory control is the means by which material of the correct quality and quantity is made available as and when it is needed with due regard to economy in the holding cost, ordering costs, setup costs, production costs, purchase costs and working capital.

Objectives of Inventory : Inventory has the following main objectives:

- To supply the raw material, sub-assemblies, semi-finished goods, finished goods, etc. to its users as per their requirements at right time and at right price.
- To maintain the minimum level of waste, surplus, inactive, scrap and obsolete items.
- To minimize the inventory costs such as holding cost, replacement cost, breakdown cost and shortage cost.
- To maximize the efficiency in production and distribution.
- To maintain the overall inventory investment at the lowest level
- To treat inventory as investment which is risky? For some items, investment may lead to higher profits and for others less profit.

Inventory Conversion Diagram: The socks at input are called raw materials whereas thestocks at the output are called products. The stocks at the conversion process may be called finished or semi-finished goods or sometimes may be raw material depending on the requirement of the product at conversion process, where the input and output are based on the market situations of uncertainty, it becomes physically impossible and economically impractical for each stock item to arrive exactly where it is required and when it is required.

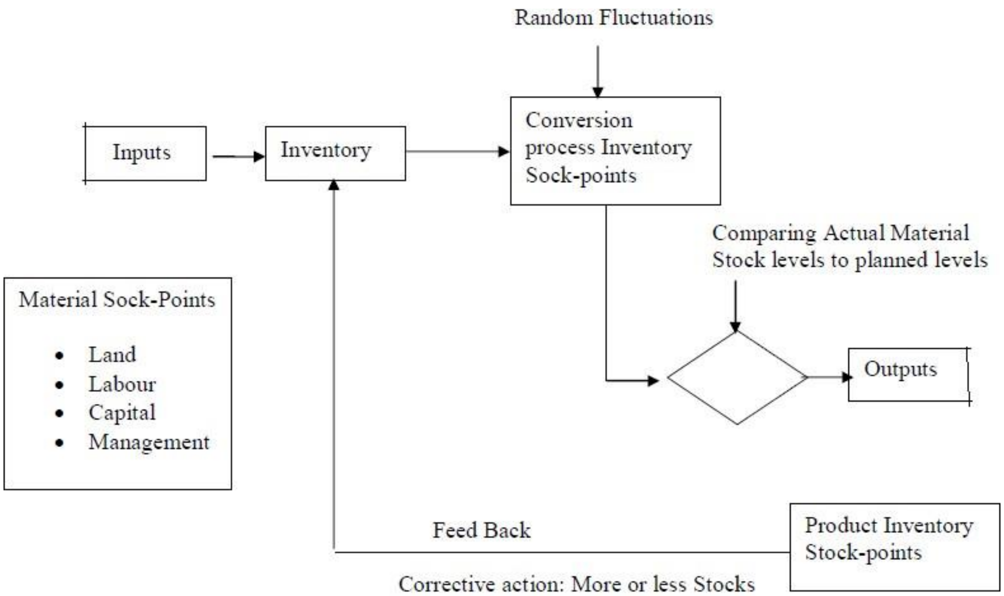


Fig: Materials Conversion Process

Role of Inventory:

Inventories play an essential and pervasive role in any organization because they make it possible:

- To meet unexpected demand
- To achieve return on investment
- To order largest quantities of goods, components or materials from the suppliers at Advantageous prices
- To provide reasonable customer service through supplying most of the requirements from Stock without delay
- To avoid economically impractical and physically impossible delivering/getting right Amount of stock at right time of required
- To maintain more work force levels
- To facilitate economic production runs
- To advantage of shipping economies
- To smooth seasonal or critical demand
- To facilitate the intermittent production of several products on the same facility
- To make effective utilization of space and capital
- To meet variations in customer demand
- To take the advantage of price discount
- To hedge against price increases
- To discount quantity

Basic Functions of Inventory :

The important basic function of inventory is

- Increase the profitability- through manufacturing and marketing support. But zero inventory manufacturing- distribution system is not practically possible, so it is important to remember that each rupee invested in inventory should achieve a specific goal.

The other inventory basic functions are

- Geographical Specialization
- Decoupling
- Balancing supply and demand and
- Safety stock

Factors affecting inventory

1.) **Inventory or stock cost:-**

There are several:

i) Purchase/Production cost– cost of purchasing a unit of item

ii) Ordering/Acquisition/Set-up cost – costs related to acquisition of purchased items i.e. those of getting an item to a firm's store e.g. transport, loading and off-loading, inspection.

iii) Inventory carrying/ holding costs – costs associated with holding a given level of inventory e.g. warehousing, spoilage, security, pilferage, administrative, insurance, depreciation.

iv) Stock-out cost/ shortage costs – incurred due to a delay in meeting demand or inability to meet demand at all because of shortage of stock loss of future sales, cost associated with future replenishment.

2. **Order cycle** – the time period between placements of 2 successive orders.

3. **Lead time** – time between placing an order and actual replenishment of item. Also referred to as procurement time.

4. **Time horizon** – this is the period over which the inventory level will be controlled.

5. **Maximum stock** – the level beyond which stocks should not be allowed to rise.

6. **Minimum stock level/buffer stock/safety stock** – level below which stock should not be allowed to fall. It is the additional stock needed to allow for delay in delivery or for any higher than expected demand that may arise due to lead time.

7. **Reorder level** – point at which purchased order must be sent to supplier for the supply of more stock. The level of stock at which further replenishment order should be placed.

8. **Reorder quantity** – the quantity of the replacement order.

ROP (Reorder Point) = Daily Demand X Lead Time

$$ROP = D/T \times T_L$$

Note that Demand is on daily basis

9. **Average stock level**

$$\text{Average stock level} = \frac{\text{Minimum stock level} + \text{Maximum stock level}}{2}$$

10. **Physical stock** – no. of items physically in stock at any given time.

11. **Stock replenishment** – rate at which items are added to the inventory.

12. **Free stock** – the physical stock plus the outstanding replenishment orders minus the unfulfilled requirements.

13. **Economic order quantity (EOQ)** – the quantity at which the cost of having stocks is minimum.

14. **Economic batch quantity (EBQ)** – quantity of stock within the enterprise. Company orders from within its own warehouses unlike in EOQ where it is ordered from elsewhere.

15. Demand:-

- Customer's demand, size of demand, rate of demand and pattern of demand is important
- Size of demand = no. of items demanded per period
- Can be deterministic (Static or dynamic) or probabilistic (governed by discrete or continuous probability distribution)
- The rate of demand can be variable or constant
- Pattern reflects items drawn from inventory -instantaneous (at beginning or end) or gradually at uniform rate

There are four major elements of inventory costs that should be taken for analysis, such as
(1) Item cost, Rs. C/item.
(2) Ordering cost, Rs. Co/order.
(3) Holding cost Rs. Ch/item/unit time.
(4) Shortage cost Rs. Cs/item/Unit time.

(1) Item Cost (C)
This is the cost of the item whether it is manufactured or purchased. If it is manufactured, it includes such items as direct material and labor, indirect materials and labor and overhead expenses. When the item is purchased, the item cost is the purchase price of 1 unit. Let it be denoted by Rs. C per item.

(2) Purchasing or Setup or Acquisition or Ordering Cost (Co)
Administrative and clerical costs are involved in processing a purchase order, expediting, follow up etc., It includes transportation costs also. When a unit is manufactured, the unit set up cost includes the cost of labor and materials used in the set up and set up testing and training costs. This is denoted by Rs. Co per set up or per order

Inventory holding cost (Ch): If the item is held in stock, the cost involved is the item carrying or holding cost. Some of the costs included in the unit holding cost are
(1) Taxes on inventories
(2) Insurance costs for inflammable and explosive items,
(3) Obsolescence,
(4) Deterioration of quality, theft, spillage and damage to times,
(5) Cost of maintaining inventory records.
This cost is denoted by Rs. Ch/item/unit time. The unit of time may be days, months, weeks or years.

Shortage Cost (Cs): The shortage cost is due to the delay in satisfying demand (due to wrong planning); but the demand is eventually satisfied after a period of time. Shortage cost is not considered as the opportunity cost or cost of lost sales. The unit shortage cost includes such items as,
(1) Overtime requirements due to shortage,
(2) Clerical and administrative expenses.
(3) Cost of expediting.
(4) Loss of goodwill of customers due to delay.
(5) Special handling or packaging costs.
(6) Lost production time.
This cost is denoted by Rs. Cs per item per unit time of shortage.

The basic deterministic inventory models

1. EOQ Model with Uniform Demand
2. EOQ Model with Different rates of Demands in different cycles
3. EOQ Model with Shortages (backorders) allowed
4. EOQ Model with Uniform Replenishment

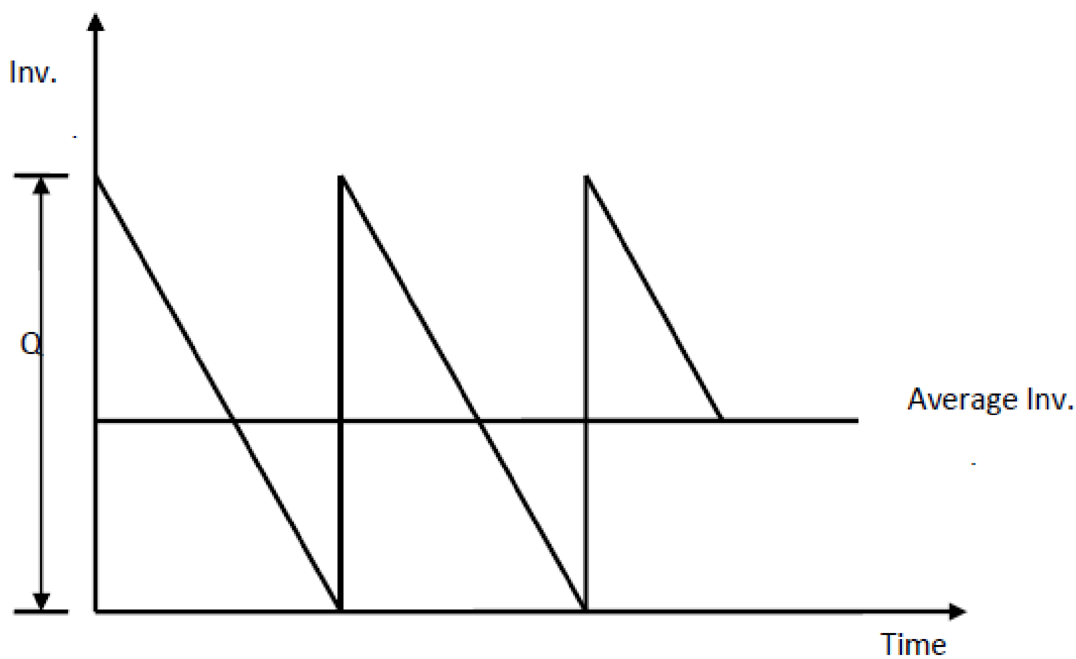
Notations used:-
Q = number of units per order
Q* = economic order quantity or optimal no. of units per order to minimize total cost
D = annual demand requirement (units per year)
C = cost of 1 unit of item
C0 = ordering (preparation or set-up) cost of each order
Ch = Cc = holding or carrying cost per unit per period of time
T = length of time between two successive orders
N = no. of orders or manufacturing runs per year
TC = Total Inventory cost
The optimal order quantity (EOQ) is at a point where the ordering cost = holding cost

Model 1- EOQ Model with Uniform Demand

Policy: Whenever the inventory level is 0, order Q items
Objective: Choose a Q that will minimize total Inventory Cost
The behavior of inventory at hand with respect to time is illustrated below:
1. No stock-out is allowed.
2. Quantity discounts are not allowed – purchase price is constant.
3. Lead time is known and fixed.
This is the ordering quantity which minimizes the balance of cost between inventory holding cost and ordering costs
.

It is based on the following assumptions:

1. A known constant stock holding cost.
2. A known constant ordering cost.
3. The rate of demand is known (is deterministic).
4. A known constant price per unit.
5. Inventory replenishment is done instantaneously
6. No stock-out is allowed.
7. Quantity discounts are not allowed – purchase price is constant.
8. Lead time is known and fixed.



1. **Annual ordering cost**

$$Annual\ ordering\ cost = (no.\ of\ orders\ placed\ per\ year) \times (ordering\ cost\ per\ order)$$

$$= \left(\frac{Annual\ Demand}{no.\ of\ units\ in\ each\ order} \right) \times (order\ cost\ per\ order)$$

$$= \frac{D}{Q} \times C_o \cdots\cdots\cdots\cdots\cdots (1)$$

2. **Annual holing (or carrying) cost**

$$Annual\ ordering\ cost = (Average\ inventory\ level) \times (carrying\ cost\ per\ order)$$

$$= \frac{Q}{2} \times C_h \cdots\cdots\cdots\cdots\cdots (2)$$

3. **Equating (1) and (2) above**

Since the minimum TC occurs at the point where the ordering cost and the inventory carrying costs are equal, we equate the 2 equations above.

$$\frac{D}{Q} \times C_o = \frac{Q}{2} \times C_h$$

Solve for Q

$$2DC_o = Q^2 C_h$$

$$Q^2 = \frac{2DC_o}{C_h}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

1. Inventory holding or carrying costs are often expressed as annual percentage(s) of the unit cost or price.

C_o or C_h as % of unit cost or price

I = annual inventory carrying charge (cost) as 1% of price

C_h = **IC** where C is the unit price of inventory item

$$EOQ = Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

2. Total cost is sum of annual C_h and annual ordering cost.

$$TC = \frac{D}{Q}.C_o + \frac{Q}{2}.C_h$$

Put value of Q* in TC,

$$TVC = \sqrt{2DC_oC_h}$$

Problem:

A supplier is required to deliver 20000 tons of raw materials in one year to a large manufacturing organization. The supplier maintains his go-down to store the material received from various resources. He finds that cost of inventory holding is 30 paisa per ton per month. His cost for ordering the material is Rs. 400. One of the conditions of the supplier contract from the manufacturing organization is that the contract will be terminated in the event of supply not being maintained as a schedule. Determine (1) in what lot size is the supplier should produce the material for minimum total associated cost of inventory? (2) At what time interval should he procure the material? It may be assume that replacement of inventory is instantaneous

Solution:

Given Date

D = 20000 tons

T = 12 months

C_h = Rs. 0.30 per tons per months

C_o = Rs. 400

(1) Economic order quantity

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(12000)(400)}{0.30 \times 12}}$$

$$Q^* = 2108 \; tons$$

(2) Tine interval

$$t_{co} = \frac{Q^*}{D}$$

$$= \frac{2108}{20000}$$

$$= 1.26 \text{ month}$$

Problem :

In the above example, if there is (i) 10 per cent increase in holding cost or (ii) 10 percent increase in ordering cost, in each case determine the optimal lot size and corresponding minimum total expected cost of inventory. Comment the result.

Ans:-

(i) $C_h' = 1.1\, C_h$

$$EOQ = \sqrt{\frac{2DC_o}{C_h'}}$$

$$= \sqrt{\frac{2(12000)(400)}{1.1 \times 0.30 \times 12}}$$

$$Q^* = 2010\ tons$$

$$TAC = \sqrt{2C_h' C_o D}$$

$$= \sqrt{2 \times 1.1 \times 0.30 \times 400 \times 12000}$$

$$= 7960 \text{ Rs.}$$

(ii) $C_o' = 1.1\, C_o$

$$EOQ = \sqrt{\frac{2DC_o'}{C_h}}$$

$$= \sqrt{\frac{2(12000)(400)(1.1)}{0.30 \times 12}}$$

$$Q^* = 2211\ tons$$

$$TAC = \sqrt{2C_h C_o' D}$$

$$= \sqrt{2 \times 0.30 \times 1.1 \times 400 \times 12000}$$

$$= 7960 \text{ Rs.}$$

Problem :

A certain item costs Rs. 250 per ton. The monthly requirement is 5 tons and each time the stock is replenished, there is an order cost of Rs. 120. The cost of carrying inventory has been estimated at 10% of the value of the stock per year. What is the optimal order quantity? If lead time is 3 months, determine the re order point. At what intervals the order should be placed?

Solution: -

Given Data

C = Rs. 250 per ton

Co = Rs. 120

C_h = 250×0.1 = Rs. 25 per ton per year

D = 5×12 = 60 tons

T_L = 3 months

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$
$$= \sqrt{\frac{2(60)(120)}{25}}$$
$$Q^* = 24\ tons$$

$$t_{co} = \frac{Q^*}{D}$$
$$= \frac{24}{60}$$
$$= 0.4 \text{ year or } 4.8 \text{ months}$$

$$Q_R = \frac{D}{T} \times T_L$$
$$= \frac{60}{12} \times 3$$
$$= 15 \text{ tons}$$

Problem :

A manufacturer has to supply his customers with 1200 units of his product per annum. Theinventory carrying cost amounts to ₹ 1.2 per unit. The set-up cost per run is ₹ 160. Find:

i) EOQ

ii) Minimum average yearly cost

iii) Optimum no of orders per year

iv) The optimum time between orders (optimum period of supply per optimum order)

Solution:

i) Economic order quantity

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$
$$= \sqrt{\frac{2(1200)(160)}{1.2}}$$
$$= 565.69 \text{ or } 566\ units$$

ii) Minimum average yearly cost

$$TC = \frac{D}{Q}.C_o + \frac{Q}{2}.C_h$$

$$TC(Q^*) = \frac{DC_o}{Q^*} + \frac{Q^* C_h}{2}$$

$$= \frac{1200(160)}{566} + \frac{566(1.2)}{2}$$

$$= 339.22 + 339.6$$

$$= \text{Rs } 678.82 \text{ or Rs } 679$$

iii) Optimum no. of orders per year (N^*)

$$N^* = \frac{Demand}{EOQ}$$

$$= \frac{1200}{566}$$

$$= 2.1 \text{ orders} \Rightarrow 3 \text{ orders}$$

iv) Optimum time between orders

$$T^* = \frac{\text{no. of working days in a year}}{N^*}$$

$$= \frac{365}{3}$$

$$= 122$$

Problem:

The annual demand per item is 6400 units. The unit cost is ₹ 12 and the inventory carrying charges 25% per annum. If the cost of procurement is ₹ 300 determine:

i) EOQ
ii) No. of orders per year
iii) Time between 2 consecutive orders
iv) Optimum cost

Ans:-

i) EOQ

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2(6400)(300)}{(0.25)(12)}}$$

$$= 1131 \text{ units}$$

ii) N^*

$$N^* = \frac{Demand}{EOQ}$$

$$= \frac{6400}{1131}$$

$$= 5.65 \text{ orders} \Rightarrow 6 \text{ orders}$$

iii) Time between 2 consecutive orders

$$T^* = \frac{no.\ of\ working\ days\ in\ a\ year}{N^*}$$

$$= \frac{365}{5.65}$$

$$= 64.60$$

(OR)

$$T^* = \frac{EOQ}{Demand} \times 12\ months$$

$$= \frac{1131}{6400} \times 12\ months$$

$$= 2\ months\ 4\ days$$

Model 2- EOQ Model with Different Rates of Demand

Assumptions of this model are same as those of model 1 except

Demand rate is different in different cycles. The total demand D is specified as demand during time horizon T

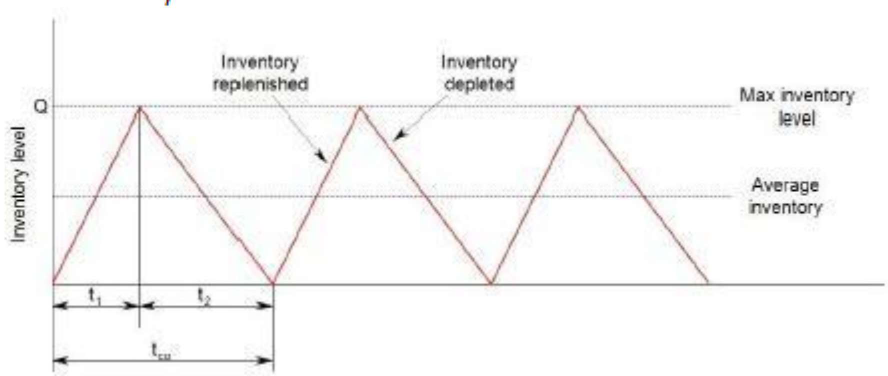
$$Holding\ Cost = \frac{Q}{2} \times (\frac{d-r}{d})$$

$$Order / set\ \text{up cost} = \frac{D \times C_o}{Q}$$

$$EOQ = Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}}$$

$$TVC = \sqrt{2rC_oC_h(\frac{d-r}{d})}$$

$$t_{co} = \frac{Q^*}{r}$$



Problem:

A manufacturing company needs 4000 units of material every month. The delivery system from the supplier is so scheduled that once delivery commences the materials is received at the rate of 6000 units per month. The cost of processing purchase order is Rs. 600 and the inventory carrying cost is 30 paisa per unit per month. Determine the optimal lot size and interval at which the order is to be placed. What is maximum inventory during a cycle?

Solution:-

Given Data

Co = Rs. 600

Ch = Rs. 0.30 per unit per month

d = 6000 units per months

r = 4000 units per months

Optimal lot size

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}}$$

$$= \sqrt{\frac{2 \times 4000 \times 600}{0.30}} \times \sqrt{\frac{6000}{6000-4000}}$$

$$= 6928 \text{ units}$$

Interval time

$$t_{co} = \frac{Q^*}{r}$$

$$= \frac{6928}{4000}$$

$$= 1.732 \text{ months}$$

Maximum inventory Q_{max}

$$Q_{max} = Q^* - rt_1$$

$$\text{Where, } t_1 = \frac{Q^*}{d}$$

$$= \frac{6928}{6000}$$

$$= 1.154 \text{ months}$$

$$Q_{max} = 6928 - 4000 \times 1.154$$

$$= 2309.33 \text{ units}$$

Problem:
The demand for a certain item is 150 units per week. No shortages are to be permitted. Holding cost is 5 paisa per unit per week. Demand can be met either by manufacturing or purchasing. With each source the data are as follows

	Manufacture	Purchase
Item cost Rs./ Unit	10.50	12
Set up/ Ordering cost Rs. / Order or set up	90	20
Replenishment rate units / week	260	Infinite
Lead time in weeks	4	10

Determine (a) the minimum cost procurement source and its economic advantage over its alternative resource, (b) E.O.Q. or E.B.Q. as per the source selected, (c) the minimum procurement level (Re-order point)

Solution:-
For manufacture

Given Data
Co = Rs. 90
Ch = Rs. 0.05 per unit per month
d = 260 units per weeks
r = 150 units per weeks
T_L = 4 week
C = Rs. 10.50 per unit

$$TVC = \sqrt{2rC_oC_h(\frac{d-r}{d})} + C \times r$$

$$= \sqrt{2 \times 150 \times 90 \times 0.05 \times (\frac{260-150}{260})} + 10.50 \times 150$$

$$= 1598.9 \text{ Rs. per week}$$

For purchase

Given Data
Co = Rs. 20
Ch = Rs. 0.05 per unit per month
r = 150 units per weeks
TL = 4 week

C = Rs. 12 per unit

$$TVC = \sqrt{2rC_oC_h} + C \times r$$

$$= \sqrt{2 \times 150 \times 20 \times 0.05} + 12 \times 150$$

$$= 1817.32 \text{ Rs. per week}$$

(a) Minimum TC is 1598.9 per week for manufacture
(b) E.B.Q

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}}$$

$$= \sqrt{\frac{2 \times 90 \times 150}{0.05}} \times \sqrt{\frac{260}{260-150}}$$

$$= 1129.76 \text{ units}$$

(c) Re-order point

$$Q_R = r \times T_L$$

$$= 150 \times 4$$

$$= 600 \text{ units}$$

Model 3- EOQ Model with Shortages (backorders) allowed

Assumptions of this model are same as those of model 1 except Shortages is allowed.

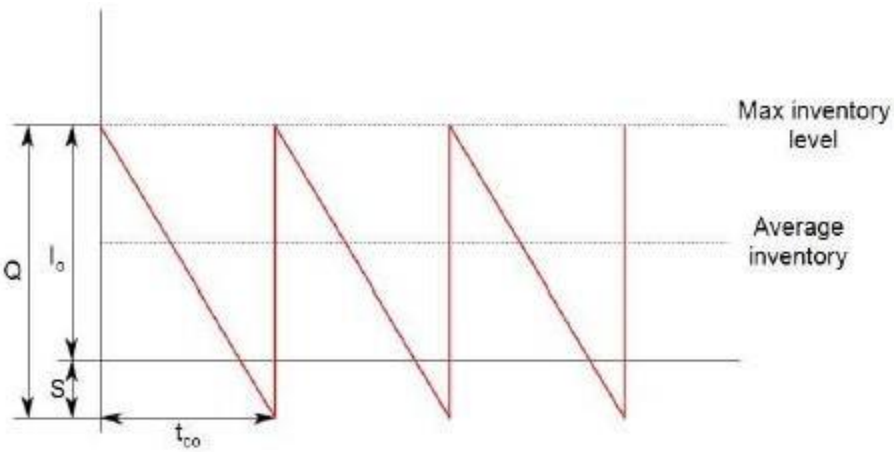
$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$TVC = \sqrt{2rC_oC_h(\frac{C_s}{C_h + C_s})}$$

$$t_{co} = \frac{Q^*}{r}$$

Initial stock I_o

$$I_o = Q^* \times (\frac{C_s}{C_h + C_s})$$



Problem:

A tractor manufacturing company has entered in to a contract with M/s Auto Diesel for delivering 30 engines per day. M/s Auto Diesel has committed that for every day's delay in delivery; there will be penalty of delayed supply at the rate of Rs. 100 per engine per day. M/s Auto Diesel has the inventory holding cost of Rs. 600 per engine per month. Assume replenishment of engines as instantaneous and ordering cost as Rs. 15000. What should be initial inventory level and what should be ordering quantity for minimum associated cost of inventory? At what interval procurement should be made?

Solution:-

Given Data

Co = Rs. 15000

Ch = Rs. 600 per engine per month

Cs = 100*30 =3000 per engine per month

r = 30*30 = 900 engine per month

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$= \sqrt{\frac{2 \times 900 \times 15000}{600}} \times \sqrt{\frac{600+3000}{3000}}$$

$$= 233 \text{ engines}$$

$$I_o = Q^* \times \left(\frac{C_s}{C_h + C_s}\right)$$

$$= 233 \times \left(\frac{3000}{600+3000}\right)$$

$$= 195 \text{ engines}$$

$$t_{co} = \frac{Q^*}{r}$$

$$= \frac{233}{30}$$

$$= 7.76 \text{ days}$$

Problem:

In above example, find out optimum order quantity if shortage is not permitted. Compare this with the value of obtained in above example and comment on the result.

Ans:-

$$Q^* = \sqrt{\frac{2rC_o}{C_h}}$$

$$= \sqrt{\frac{2 \times 900 \times 15000}{600}}$$

$$= 212 \text{ engines}$$

$\Rightarrow$ If shortage is not permitted, EOQ is reducing 233 engines per order to 212 engines per order.

Model 4- EOQ Model with Uniform Replenishment

Assumptions of this model are same as those of model 1 except Demand is variable and Shortages is allowed.

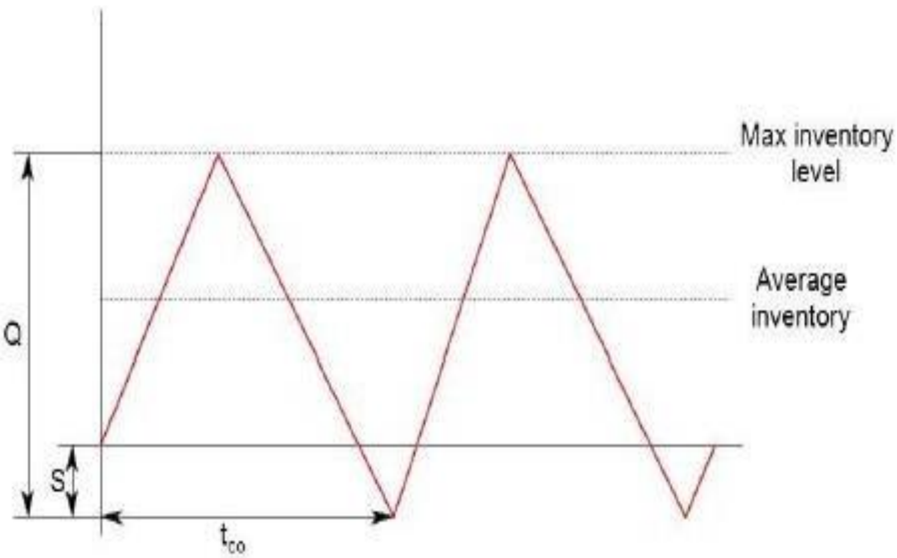
$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$TVC = \sqrt{2rC_oC_h(\frac{d-r}{d})(\frac{C_s}{C_h + C_s})}$$

$$t_{co} = \frac{Q^*}{r}$$

Initial stock I_o

$$I_o = Q^* \times (\frac{C_s}{C_h + C_s}) \times (\frac{d-r}{d})$$



Problem :

The demand for an item in a company is 18000 units per year and the company can produce the item at a rate of 3000 units per month. The set up cost is Rs. 500 per set up and the annual inventory holding cost is estimated at 20 percent of the investment in average inventory. The cost of one unit short is Rs. 20 per year. Determine, (i) Optimal production batch quantity, (ii) Optimum cycle time and production time, (iii) Maximum inventory level in the cycle, (iv) Maximum shortage permitted and (v) Total associated cost per year. The cost of the items is Rs. 20 per unit.

Solution:-

Given Data Co = Rs. 500

Ch = 0.20*20 = Rs. 4 per unit per year Cs = Rs. 20 per unit per year r = 18000 unit per year

d = 3000*12 = 36000 unit per year

$$Q^* = \sqrt{\frac{2rC_o}{C_h}} \times \sqrt{\frac{d}{d-r}} \times \sqrt{\frac{C_h + C_s}{C_s}}$$

$$= \sqrt{\frac{2 \times 18000 \times 500}{4}} \times \sqrt{\frac{36000}{36000-18000}} \times \sqrt{\frac{4+20}{20}}$$

$$= 3286 \text{ units}$$

$$t_{co} = \frac{Q^*}{r}$$

$$= \frac{3286}{18000}$$

$$= 0.18255 \text{ year}$$

$$= 2.19 \text{ months}$$

$$t_{po} = \frac{Q^*}{d}$$

$$= \frac{3286}{36000}$$

$$= 0.091277 \text{ year}$$

$$= 1.09 \text{ months}$$

$$I_o = Q^* \times (\frac{C_s}{C_h + C_s}) \times (\frac{d-r}{d})$$

$$= 3286 \times (\frac{20}{4+20}) \times (\frac{36000-18000}{36000})$$

$$= 1369 \text{ units}$$

$$S = Q^* \times (\frac{C_h}{C_h + C_s}) \times (\frac{d-r}{d})$$

$$= 3286 \times (\frac{4}{4+20}) \times (\frac{36000-18000}{36000})$$

$$= 273.88 \text{ units}$$

$$TC = \sqrt{2rC_oC_h(\frac{d-r}{d})(\frac{C_s}{C_h+C_s})} + C \times r$$

$$TC = \sqrt{2 \times 18000 \times 500 \times 4 \times (\frac{36000-18000}{36000})(\frac{20}{4+20})} + 20 \times 18000$$

$$= 365477.22$$

Model 5- EOQ Model with Quantity Discounts :

Quantity discounts occur in numerous situations where suppliers provide an incentive for large order quantities by offering a lower purchase cost when items are ordered in larger lots or quantities. In this section we show how the EOQ model can be used when quantity discounts are available.

EOQ without discounts

$$Q^* = \sqrt{\frac{2rC_o}{C_h}}$$

- EOQ with discounts

$$Q_o^* = \frac{(C_d \times r + Q^* \times i \times C)}{i(C - C_d)}$$

- Max Net Saving with discounts

$$X_{max} = \frac{(C_d \times r + Q^* \times i \times C)^2}{2 \times i \times r(C - C_d)} - C_o$$

Problem:

A wholesale dealer in bearings purchases 30000 bearings annually at intervals and order size suitable to him. The price is Rs. 150 per bearing. The manufacturing company offers the dealer a discount of Rs. 7 per bearing for the order size larger than earlier. The reorder cost is Rs. 40 and the inventory carrying cost amounts to 20 percent of the investment in purchase price. Decide the optimum order size for special discount offer purchase and the maximum benefit he can derive from this order.

Solution:-

Given Data Co = Rs. 40 Ch = 0.20*150 = Rs. 30 per unit per year r = 30000 unit per year
C = Rs 150 per unit Cd = Rs 7 per unit i = 0.20

$$Q^* = \sqrt{\frac{2rC_o}{C_h}}$$

$$= \sqrt{\frac{2 \times 30000 \times 40}{30}}$$

= 283 units

$$Q_o^* = \frac{(C_d \times r + Q^* \times i \times C)}{i(C - C_d)}$$

$$= \frac{(7 \times 30000 + 283 \times 0.2 \times 150)}{0.2(150 - 7)}$$

= 7640 Units

$$X_{max} = \frac{(C_d \times r + Q^* \times i \times C)^2}{2 \times i \times r(C - C_d)} - C_o$$

$$= \frac{(7 \times 30000 + 283 \times 0.2 \times 150)^2}{2 \times 0.2 \times 30000(150 - 7)} - 40$$ = 27779

Model 6- Probabilistic Inventory Models

The inventory models that we have discussed thus far have been based on the assumption that the demand rate is constant and deterministic throughout the year. We developed minimum-cost order quantity and reorder-point policies based on this assumption. In situations where the demand rate is not deterministic, models have been developed that treat demand as probability distribution. In this section we consider a single-period inventory model with probability demand. The single-period inventory model refers to inventory situations in which one order is placed for the product; at the end of the period, the product has either sold out, or there is a surplus of unsold items that will be sold for a salvage value. The single-period inventory model is applicable in situations involving seasonal or perishable items that cannot be carried in inventory and sold in future periods. Seasonal clothing (such as bathing suits and winter coats) is typically handled in a single-period manner. In these situations, a buyer places one preseason order for each item and then experiences a stock out or hold a clearance sale on the surplus stock at the end of the season. No items are carried in inventory and sold the following year. Newspapers are another example of a product that is ordered one time and is either sold or not sold during the single period. While newspapers are ordered daily, they cannot be carried in inventory and sold in later periods. Thus, newspaper orders may be treated as a sequence of single-period models; that is, each day or period is separate, and a single-period inventory decision must be made each period (day). Since we order only once for the period, the only inventory decision we must make is how much of the product to order at the start of the period. Because newspaper sales are an excellent example of a single-period situation, the single-period inventory problem is sometimes referred to as the newsboy problem.

Optimum stock level

$$P_r = \frac{C_s}{C_s + C_h}$$

Problem:

A large industrial campus has decided to have its diesel generator system for street lighting, security illumination and round the clock process systems. The generator needs a tailor made for each other control unit which cost Rs. 18000 per number when ordered with the total equipment of diesel generator. A decision needs to be taken whether additional numbers of this unit should be ordered along with equipment, and if so, how many units should be ordered? These control units, though tropicalized and considered quite reliable, are known to have failed from time to time and history of failures of similar equipment give the following probability of failure. It is found that if the control unit fails, the entire generator system comes to a grinding halt. When control unit fails and a spare unit is not available it is estimated that the cost rush order procurement, including the associated cost of the downtime is Rs. 50000 per unit. Considering that any investment in inventory is the cost of inventory, decide how many spare units should be ordered along with the original order. Determine total associated cost for each no. of spare unit

No. of units having failed and hence No. of spare Required	0	1	2	3	4	5	6
Probability	0.6	0.2	0.1	0.05	0.03	0.02	0

Ans:-

⇒ Let us consider the elementary approach as the population of demand varies only from 0 to 5.

⇒ I = 0,

$$TAC = C_s \times 1 \times P_1 + C_s \times 2 \times P_2 + C_s \times 3 \times P_3 + C_s \times 4 \times P_4 + C_s \times 5 \times P_5$$

$$= 50000(0.2 + 2 \times 0.1 + 3 \times 0.05 + 4 \times 0.03 + 5 \times 0.02)$$

$$= 38500$$

$\Rightarrow$ I = 1,

$$TAC = C_h \times 1 \times P_0 + C_s \times 0 \times P_1 + C_s \times 1 \times P_2 + C_s \times 2 \times P_3 + C_s \times 3 \times P_4 + C_s \times 4 \times P_5$$

$$= 18000 \times 0.6 + 50000 \times 0.1 + 50000 \times 2 \times 0.05 + 50000 \times 3 \times 0.03 + 50000 \times 4 \times 0.02$$

$$= 29300$$

$\Rightarrow$ I = 2,

$$TAC = C_h \times 2 \times P_0 + C_h \times 1 \times P_1 + C_s \times 0 \times P_2 + C_s \times 1 \times P_3 + C_s \times 2 \times P_4 + C_s \times 3 \times P_5$$

$$= 18000 \times 2 \times 0.6 + 18000 \times 0.2 + 50000 \times 1 \times 0.05 + 50000 \times 2 \times 0.03 + 50000 \times 3 \times 0.02$$

$$= 33700$$

$\Rightarrow$ I = 3,

$$TAC = C_h \times 3 \times P_0 + C_h \times 2 \times P_1 + C_h \times 1 \times P_2 + C_s \times 0 \times P_3 + C_s \times 1 \times P_4 + C_s \times 2 \times P_5$$

$$= 18000 \times 3 \times 0.6 + 18000 \times 2 \times 0.2 + 18000 \times 1 \times 0.05 + 50000 \times 1 \times 0.03 + 50000 \times 2 \times 0.02$$

$$= 44900$$

$\Rightarrow$ I = 4,

$$TAC = C_h \times 4 \times P_0 + C_h \times 3 \times P_1 + C_h \times 2 \times P_2 + C_h \times 1 \times P_3 + C_s \times 0 \times P_4 + C_s \times 1 \times P_5$$

$$= 18000 \times 4 \times 0.6 + 18000 \times 3 \times 0.2 + 18000 \times 2 \times 0.05 + 18000 \times 1 \times 0.03 + 50000 \times 1 \times 0.02$$

$$= 59500$$

$\Rightarrow$ I = 5,

$$TAC = C_h \times 5 \times P_0 + C_h \times 4 \times P_1 + C_h \times 3 \times P_2 + C_h \times 2 \times P_3 + C_h \times 1 \times P_4 + C_s \times 0 \times P_5$$

$$= 18000 \times 5 \times 0.6 + 18000 \times 4 \times 0.2 + 18000 \times 3 \times 0.05 + 18000 \times 2 \times 0.03 + 18000 \times 1 \times 0.02$$

$$= 76140$$

Cumulative Probability Table

No. of units	$\sum_0^s P_r$
0	0.6
1	0.8
2	0.9
3	0.95
4	0.98
5	1.00
6	1.00

$$P_r = \frac{C_s}{C_s + C_h} = \frac{50000}{50000 + 18000} = 0.7352$$

As I = 1, total associated cost is minimum, 1 spare units should be ordered along with the original order. □□Pr is between 0.6 and 0.8 ; $0.6 \le 0.7352 \le 0.8$

Optimum stock level is 1.

Problem: In the above problem as regular purchase price of control unit is almost one third of the estimated rush order associated cost of one unit. The management decides to buy two spare units with the first order. Having decided that, the management would like to know for what range of actual values of shortage cost, the decision is justified

Ans:-

⇒ For I = 2 to be optimum

$$P_{r1} < \frac{C_s}{C_s + C_h} < P_{r2}$$

$$\Rightarrow P_{r1} < \frac{C_s}{C_s + C_h}$$

$$0.8 < \frac{C_s}{C_s + 6000}$$

$$24000 < C_s$$

$$\Rightarrow \frac{C_s}{C_s + C_h} < P_{r2}$$

$$\frac{C_s}{C_s + 6000} < 0.9$$

$$C_s < 54000$$

⇒ Value of C_s is between 24000 and 54000

Problem -

Probabilistic demand of sweets in a large chain of sweet marts is rectangular between 1000 kg and 1400 kg. Profit per kg of fresh sweet sold is Rs. 14.70. If sweet is not sold fresh, next day it can be sold at a loss of Rs. 2.30 per kg. Determine the optimum stock to have fresh sweet on hand every day.

Ans:- Given Data Co = Rs. 14.70 per kg Ch = Rs 2.30 per kg Range = 1400-1000 = 400

$$f(r) = \frac{1}{Range} = \frac{1}{400}$$

$$\int_{1000}^{I_0} f(r)\,dr = \frac{C_s}{C_s + C_h}$$

$$\int_{1000}^{I_0} \frac{1}{400}\,dr = \frac{14.70}{14.70 + 2.30}$$

$$\frac{1}{400}(I_o - 1000) = \frac{14.70}{17}$$

$$I_o = 1.346 \text{ kg}$$

Problem :

A newspaper boy buys daily papers from vendor and gets commission of 4 paisa for each paper sold. As he is always demanding large number in a lot, he has agreed to pay 3 paisa per each copy returned unsold. He has the past experience of the demand (its probability) as under.
23 (0.01), 24 (0.03), 25 (0.06), 26(0.10), 27(0.20), 28(0.25), 29(0.15), 30(0.10), 31(0.05), 32(0.05)
How many papers should he lift from vendor for minimum associated cost?

Solution:-

Given Data
Co = 4 paisa per paper
Ch = 3 paisa per paper

Cumulative Probability Table

No. of units	$\sum_{0}^{s} P_r$
23	0.01
24	0.04
25	0.10
26	0.20
27	0.40
28	0.65
29	0.80
30	0.90
31	0.95
32	1.00

$$P_r = \frac{C_s}{C_s + C_h}$$
$$= \frac{4}{4+3}$$
$$= 0.571$$

$\Rightarrow$ P_r is between 0.6 and 0.8
$0.4 < 0.571 < 0.65$
$\Rightarrow$ Optimum stock level is 28 newspapers.

ABCAnalysys:

ABC analysis is an inventory categorization method which consists in dividing items into three categories (A, B, C):

- A being the most valuable items,
- B bieng Inter class item
- C being the least valuable ones.

This method aims to draw managers' attention on the critical few (A-items) not on the trivial many (C-items)
The ABC approach states that a company should rate items from A to C, basing its ratings on the following rules:

A-items are goods which annual consumption value is the highest; the top 70-80% of the annual consumption value of the company typically accounts for only 10-20% of total inventory items.
B-items are the interclass items, with a medium consumption value; those 15-25% of annual consumption value typically accounts for 30% of total inventory items.
C-items are, on the contrary, items with the lowest consumption value; t
he lower 5%
of the annual consumption value typically accounts for 50% of total inventory items

	Percentage of items	Percentage value of annual usage	
Class A items	About 20%	About 80%	Close day to day control
Class B items	About 30%	About 15%	Regular review
Class C items	About 50%	About 5%	Infrequent review

Tutorial Questions

1 a)a)State Group of replacement policy

b) The following failure rates have been observed for a certain type of light bulbs

10. End of week	11. Probability of failure date
12. 1	13. 0.05
14. 2	15. 0.13
16. 3	17. 0.25
18. 4	19. 0.43
20. 5	21. 0.68
22. 6	23. 0.88
24. 7	25. 0.96
26. 8	27. 1.00

The cost of replacing an individual failed bulb is Rs.1.25.the decision is made to replace all bulbs simultaneously at fixed intervals and also to replace individual bulbs as they fall in service. If the cost of group replacement is 30 paise per bulb, what is the best interval between group replacements? At what group replacement price per bulb would a policy of strictly individual replacement become preferable to the adopted policy?

2. A firm is considering the replacement of a machine, whose cost price is Rs.12,200 and its shop value is Rs.200. From experience the running (maintenance and operating) costs are found to be as follows.

Year	1	2	3	4	5	6	7	8
Ruuning cost	200	500	800	1200	1800	2500	3200	4000

3. a) When should the machine be replaced?

b) Find the most economic batch quantity of a product on machine if the production rate of the item on the machine is 300 pieces per day and the demand is uniform at the rate of 150 pieces/day. The set up Cost is Rs.300 per batch and the cost of holding one item in inventory is Rs.0.81/per day. How will the batch quantity vary if the machine production n rate was infinite?

4. A salesman has to visit five cities A,B,C,D,E. The intercity distances are tabulated below

9.	10. A	11. B	12. C	13. D	14. E
15. A	16. -	17. 12	18. 24	19. 25	20. 15
21. B	22. 6	23. -	24. 16	25. 18	26. 7
27. C	28. 10	29. 11	30. -	31. 18	32. 12
33. D	34. 14	35. 17	36. 22	37. -	38. 16
39. E	40. 12	41. 13	42. 23	43. 25	44. -

Find the shortest route covering all the cities.

5. The data collected in running a Machine the cost of which is Rs: 60,000 are

Resale value	1	2	3	4	5
Resale value(R	42,000	30,000	20,400	14,400	9,650
Cost of Spares (	4,000	4,270	4,880	5,700	6,800
Cost of Labor	14,000	16,000	18,000	21,000	25,00

Find the time when the machine should be replaced?

Assignment Questions

1. Machine A costs of Rs:80,000. Annually operating cost are Rs:2,000 for the first years and they increase by Rs:15,000 every years (for example in the fourth year the operating cost are Rs:47,000) .Determine the least age at which to replace the machine. If the optional replacement policy is followed (

   a) What will be the average yearly cost of operating and owing the machine (Assume that the reset value of the machine is zero when replaced, and that future costs are not discounted

   b) Another machine B cost Rs:1,00,000.Annual operating cost for the first year is Rs:4,000 and they increase by Rs:7,000 every year .The following firm has a ma(chine of type A which is one year old. Should the firm replace it with B and if so when?

   (c) Suppose the firm is just ready to replace the M/c A with another M/c of the same type, just the the firm gets an information that the M/c B will become available in a year .What should firm do?

2. a) The Production department of a company required 3600 Kg, if raw material for manufacturing a particular item for a year. It has been estimated that the cost of placing an order is Rs. 36 and the cost of carrying inventory is 25% for the investment in the inventories, the price is Rs. 10/Kg. help the purchase manager to determine and ordering policy for raw material, determine optimal lot size.

   i. Define group replacement policy.

   b) A computer contains 10000 resistors. When any resistor fails, it is replaced the cost of replacing a resistor individually is Rs.1 only. If all the resistors are replaced at the same time, cost per resistor would be reduced to 35 paisa. The % of surviving resistors say S(t) at the end of month t and the P(t) the probability of failure during

   The month t is.

t	0	1	2	3	4	5	6
S(t)	100	97	90	70	30	15	0
P(t)	-	0.03	0.07	0.2	0.4	0.15	0.15

   What is the optimal replacement policy?

3. A dealer supplies you the following information with regards to a product that he deals in annual demand =10,000 units, ordering cost Rs.10/order. Price Rs.20/unit. Inventory carrying cost is 20% of the value of inventory per year. The dealer is considering the possibility of allowing some back orders to occur. He has estimated that the annual cost of back ordering will be 25% of the value of inventory

   a. What should be the optimum no of units he should buy in 1 lot?
   b. What qty of the product should be allowed to be back ordered
   c. What would be the max qty of inventory at any time of year

Would you recommend to allow backordering? If so what would be the annual cost saving by adopting the policy of back ordering.

4. .a) Purchase manager places order each time for a lot of 500 no of particular item from the available data the following results are obtained, inventory carrying 40%, ordering cost order Rs.600, cost per unit Rs.50 annual demand 1000 find out the loser to the organization due to his policy

b) What are inventory models? Enumerate various types of inventory models and describe them briefly

5. The demand for a purchased item 1000 units per month and shortages are allowed. If the unit cost is Rs. 1.50 per unit, the cost of making one purchase is Rs.600,the holding cost for one unit is Rs.2 per year and one shortage isRs.10 per year. Determine

i) The optimum purchase quantity ii)The number of orders per year iii)The optimal total yearly cost

UNIT 5

Sequencing

&

Simulation

UNIT V
Sequencing

Introduction:

The selection of an appropriate order for finite number of different jobs to be done on a finite number of machines is called **sequencing** problem. In a sequencing problem we have to determine the optimal order (sequence) of performing the jobs in such a way so that the total time (cost) is minimized.

Suppose n jobs are to be processed on m machines for successful completion of a project. Such type of problems frequently occurs in big industries. The sequencing problem is to determine the order (sequence) of jobs to be executed on different machines so that the total cost (time) involved is minimum.

Before developing the algorithm we define certain terms as M_{ij} = processing time required by i^{th} job on the j^{th} machine (i = 1 to n, j = 1 to m).

T_{ij} = idle time on machine j from the completion of $(i - 1)^{th}$ job to the start of i^{th} job.

T = elapsed time (including idle time) for the completion of all the jobs.

The problem is to determine a sequence $i_1, i_2, \ldots, i_n$, where $i_1, i_2, \ldots, i_n$ is a some permutation of the integers 1,2, ... ,n that minimizes the total elapsed time T. Each job is processed on machine M_1 and then on machine M_2 , and we say jobs functioning order is M_1M_2. Before developing the algorithm in the next section we make certain assumptions.

(i) No Machine can process more than one job at a time.

(ii) Each job once started on a machine must be completed before the start of new job.

(iii) Processing times M_{ij}'s are independent of the order of processing the jobs.

(iv) Processing times M_{ij}'s are known in advance and do not change during operation.

(v) The time required in transferring a job from one machine to other machine is negligible.

Processing of n jobs through two machines:

The simplest possible sequencing problem is that of n job two machine sequencing problem in which we want to determine the sequence in which **n**-job should be processed through two machines so as to minimize the total elapsed time T. The problem can be described as:

a) Only two machines A and B are involved;

b) Each job is processed in the order AB.

c) The exact or expected processing times A_1, A_2, A_3, --- , A_n ; B_1, B_2, B_3, --- , B_n are known and are provided in the following table.

Machine	Job(s)								
	1	2	3	--	-	i	- -	-	n
A	A_1	A_2	A_3	--	-	A_i	--	-	A_n
B	B_1	B_2	B_3	--	-	B_i	--	-	B_n

The problem is to find the sequence (or order) of jobs so as to minimize the total elapsed time T. The solution of the above problem is also known as Johnson's procedure which involves the following steps:

Step 1. Select the smallest processing time occurring in the list A_1, A_2, A_3, --- , A_n ; B_1, B_2, B_3, --- , B_n if there is a tie, either of the smallest processing times can be selected.

Step 2. If the least processing time is A_r, select the r^{th} job first. If it is B_s, do the s^{th} job last as the given order is AB

Step 3. There are now (n-1) jobs left to be ordered. Repeat steps I and II for the remaining set of processing times obtained by deleting the processing time for both the machines corresponding to the job already assigned.

Step 4. Continue in the same manner till the entire jobs have been ordered. The resulting ordering will minimize the total elapsed time T and is called the optimal sequence.

Step 5. After finding the optimal sequence as stated above find the total elapsed time and idle times on machines A and B as under:

Totalelapsedtime = The time between starting the first job in the optima sequence on machine A and completing the last job in the optimal machine B.

Idletimeon machineA = (Time when the last job in the optimal sequence on sequences is completed on machine B)- (Time when the last job in the optimal sequences is completed on machine A)

Idletimeon machineB = (Time when the first job in the optimal sequences is completed on machine A)+

$$\sum_{k=2}^{n}[(time\ when\ k^{th} job\ starts\ on\ machine\ B) - (time\ (k-1)^{st}\ job\ finshed\ on\ machine\ B)]$$

Problem:

There are nine jobs, each of which must go through two machines P and Q in the order PQ, the processing times (in hours) are given below:

Machine	Job(s)								
	A	B	C	D	E	F	G	H	I
P	2	5	4	9	6	8	7	5	4
Q	6	8	7	4	3	9	3	8	11

Find the sequence that minimizes the total elapsed time T. Also calculate the total idle time for the machines in this period.

Solution:

The minimum processing time on two machines is 2 which correspond to task A on machine P. This shows that task A will be preceding first. After assigning task A, we are left with 8 tasks on two machines

Machine	B	C	D	E	F	G	H	I
P	5	4	9	6	8	7	5	4
Q	8	7	4	3	9	3	8	11

Minimum processing time in this reduced problem is 3 which correspond to jobs E and G (both on machine Q). Now since the corresponding processing time of task E on machine P is less than the corresponding processing time of task G on machine Q therefore task E will be processed in the last and task G next to last. The situation will be dealt as

A							G	E

The problem now reduces to following 6 tasks on two machines with processing time as follows:

Machine	B	C	D	F	H	I
P	5	4	9	8	5	4
Q	8	7	4	9	8	11

Here since the minimum processing time is 4 which occurs for tasks C and I on machine P and task D on machine Q. Therefore, the task C which has less processing time on P will be processed first and then task I and task D will be placed at the last i.e., 7th sequence cell. The sequence will appear as follows:

A	C	I				D	E	G

The problem now reduces to the following 3 tasks on two machines

Machine	B	F	H
P	5	8	5
Q	8	9	8

In this reduced table the minimum processing time is 5 which occurs for tasks B and H both on machine P. Now since the corresponding time of tasks B and H on machine Q are same i.e. 8. Tasks B or H may be placed arbitrarily in the 4th and 5th sequence cells. The remaining task F can then be placed in the 6th sequence cell. Thus the optimal sequences are represented as

A	I	C	B	H	F	D	E	G

(OR)

A	1	C	H	B	F	D	E	G

Further, it is also possible to calculate the minimum elapsed time corresponding to the optimal sequencing $A \rightarrow I \rightarrow C \rightarrow B \rightarrow H \rightarrow F \rightarrow D \rightarrow E \rightarrow G$.

Job Sequence	Machine A		Machine B	
	Time In	Time Out	Time In	Time Out
A	0	2	2	8
I	2	6	8	19
C	6	10	19	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
E	37	43	55	58
G	43	50	58	61

Hence the total elapsed time for this proposed sequence staring from job A to completion of job G is 61 hours .During this time machine P remains idle for 11 hours (from 50 hours to 61 hours)and the machine Q remains idle for 2 hours only (from 0 hour to 2 hour).

Processing of n Jobs through Three Machines: The type of sequencing problem can be described as follows:

a) Only three machines A, B and C are involved;

b) Each job is processed in the prescribed order ABC

c) No passing of jobs is permitted i.e. the same order over each machine is maintained.

d) The exact or expected processing times A_1, A_2, A_3, --- , A_n ; B_1, B_2, B_3, --- , B_n and C_1, C_2, C_3, --- , C_n are known and are denoted by the following table

Our objective will be to find the optimal sequence of jobs which minimizes the total elapsed time. No general procedure is available so far for obtaining an optimal sequence in such case. However, the Johnson's procedure can be extended to cover the special cases where either one or both of the following conditions hold:

a) The minimum processing time on machine A ≥ the maximum processing time on machine B.

b) The minimum processing time on machine C ≥ the maximum processing time on machine B.

The method is to replace the problem by an equivalent problem involving n jobs and two machines. These two fictitious machines are denoted by G and H and the corresponding time G_i and H_i are defined by

$$G_i = A_i + B_i \quad \text{and} \quad B_i + C_i$$

Now this problem with prescribed ordering GH is solved by the method with n jobs through two machines, the resulting sequence will also be optimal for the original problem.

Problem :

There are five jobs (namely 1,2,3,4 and 5), each of which must go through machines A, B and C in the order ABC. Processing Time (in hours) are given below:

Machine	Job(s)								
	1	2	3	--	-	i	- -	-	n
A	A_1	A_2	A_3	--	-	A_i	--	-	A_n
B	B_1	B_2	B_3	--	-	B_i	--	-	B_n
C	C_1	C_2	C_3			C_i			C_n

Find the sequence that minimum the total elapsed time required to complete the jobs.

Solution :

Here Min $A_i = 5$; $B_i = 5$ and $C_i = 3$ since the condition of Min. $A_i \geq$ Max. B_i is satisfied the given problem can be converted into five jobs and two machines problem.

Jobs	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

The Optimal Sequence will be

2	5	4	3	1

Total elapsed Time will be

Jobs	Machine A		Machine B		Machine C	
	In	Out	In	Out	In	Out
2	0	7	7	8	8	15
5	7	12	12	15	15	22
4	12	21	21	26	26	32
3	21	27	27	31	32	37
1	27	32	32	34	37	40

Min. total elapsed time is 40 hours.

Idle time for Machine A is 8 hrs. (32-40)

Idle time for Machine B is 25 hours (0-7, 8-12, 15-21, 26-27, 31-32 and 34-40)

Idle time for Machine C is 12 hours (0-8, 22-26.)

Problems with n Jobs and m Machines

Let there be n jobs, each of which is to be processed through m machines, say M_1, M_2, --- , M_m in the order M_1, M_2, M_3, --- , M_m. Let T_{ij} be the time taken by the i^{th} machine to complete the j^{th} job. The iterative procedure of obtaining an optimal sequence is as follows:

Step I: Find (i) $\min_j(T_{1j})$, ii) $\min_j(T_{mj})$ iii) $\max_j(T_{2j}, T_{3j}, T_{4j}, ---, T_{(m-1)j})$ for j=1,2,---, n

Step II: Check whether a. $\min_j(T_{1j}) \geq \max_j(T_{ij})$ for i=2,3, --- ,m-1

Or

b. $\min_j(T_{mj}) \geq \max_j(T_{ij})$ for i=2,3,---,m-1

Step III: If the inequalities in Step II are not satisfied, method fails, otherwise, go to next step.

Step IV: Convert the m machine problem into two machine problem by introducing two fictitious machines G and H, such that

$T_{Gj} = T_{1j} + T_{2j} + --- + T_{(m-1)j}$, and $T_{Hj} = T_{2j} + T_{3j} + --- + T_{mj}$

Determine the optimal sequence of n jobs through 2 machines by using optimal sequence algorithm.

Step V: In addition to condition given in Step IV, if $T_{ij} = T_{2j} + T_{3j} + --- + T_{mj} = C$ is a fixed positive constant for all i = 1, 2, 3, , n then determine the optimal sequence of n jobs and two machines M_1 and M_m in the order M_1M_m by using the optimal sequence algorithm.

Problem:

Find an optimal sequence for the following sequencing problem of four jobs and five machines when passing is not allowed, of which processing time (in hours) is given below:

Job	Machine				
	A	B	C	D	E
1	7	5	2	3	9
2	6	6	4	5	10
3	5	4	5	6	8
4	8	3	3	2	6

Also find the total elapsed time.

Solution

Here Min. $A_i = 5$, Min. $E_i = 6$

Max. $(B_i, C_i, D_i) = 6, 5, 6$ respectively

Since Min. E_i = Max. (B_i, D_i) and Min. A_i = Max. C_i satisfied therefore the problem can be converted into 4 jobs and 2 fictitious machines G and H as follows:

Job	Fictitious Machine	
	$G_i = A_i + B_i + C_i + D_i$	$H_i = B_i + C_i + D_i + E_i$
1	17	19
2	21	25
3	20	23
4	16	14

The above sequence will be:

1 3 2 4

Total Elapsed Time Corresponding to Optimal Sequence can be obtained as follows:

	Machine A		Machine B		Machine C		Machine D		Machine E	
Job	In	Out	In	Out	In	Out	In	Out	In	Out
1	0	7	7	12	12	14	14	17	17	26
3	7	12	12	16	16	21	21	27	27	35
2	12	18	18	24	24	28	28	33	35	45
4	18	26	26	29	29	32	33	35	45	51

Thus the minimum elapsed time is 51 hours.

Idle time for machine A = 25 hours(26-51)

Idle time for machine B = 33 hours(0-7,16-18,24-26,29-51)

Idle time for machine C = 37 hours(0-12,14-16,21-24,28-29,32-51)

Idle time for machine D = 35 hours (0-14,17-21,27-28,35-51)

Idle time for machine E = 18 hours (0-17,26-27)

Simulation

Simulation:

Simulation is an experiment conducted on a model of some system to collect necessary information on the behavior of that system.
The representation of reality in some physical form or in some form of Mathematical equations are called Simulations. Simulations are imitation of reality.
For example:
1, Children cycling park with various signals and crossing is a simulation of a read model traffic system
2. Planetarium
3. Testing an air craft model in a wind tunnel.

Need for simulation:

- Consider an example of the queuing system, namely the reservation system of a transport corporation.
- The elements of the system are booking counters (servers) and waiting customers (queue).
- Generally the arrival rate of customers follow a Poisson distribution and the service time follows exponential distribution.
- Then the queuing model (M/M/1) : (GD/∞ /∞) can be used to find the standard results.

But in reality, the following combinations of distributions may exist.
1. Arrived rate does not follow Poisson distribution, but the service rate follows an exponential distribution.
2. Arrival rate follows a Poisson distribution and the service rate does not follow exponential distribution.
3. Arrival rate does not follows Poisson distribution and the service time also does not follow exponential distribution.
In each of the above cases, the standard model (M/M/1) : (G/D/∞ /∞) cannot be used. The last resort to find the
solution for such a queuing problem is to use simulation.

Advantage of simulation :

1. Simulation is Mathematically less complicated
2. Simulation is flexible
3. It can be modified to suit the changing environments.
4. It can be used for training purpose
5. It may be less expensive and less time consuming in a quite a few real world situations.

Limitations of Simulation:

1. Quantification or Enlarging of the variables maybe difficult.
2. Large number of variables make simulations unwieldy and more difficult.
3. Simulation may not. Yield optimum or accurate results.

4. Simulation are most expensive and time consuming model.
5. We cannot relay too much on the results obtained from simulation models.

Steps in simulation:

1. Identify the measure of effectiveness.
2. Decide the variables which influence the measure of effectiveness and choose those variables, which affects the measure of effectiveness significantly.
3. Determine the probability distribution for each variable in step 2 and construct the cumulative probability distribution.
4. Choose an appropriate set of random numbers.
5. Consider each random number as decimal value of the cumulative probability distribution.
6. Use the simulated values so generated into the formula derived from the measure of effectiveness.
7. Repeat steps 5 and 6 until the sample is large enough to arrive at a satisfactory and reliable decision.

Uses of Simulation:
Simulation is used for solving
1. Inventory Problem
2. Queuing Problem
3. Training Programs etc

General Purpose Languages used for Simulation:

FORTRAN: Probably more models than any other language

PASCAL; Not an universal as FORTRAN

MODULA: Many improvements over PASCAL

ADA: Department of Defense attempt at standardization

C, C++: Object-Oriented Programming Language

PSPICE: Simulation Software

MAT LAB: MATrix LABoratory : High Level Languages (Mathematical and Graphical Subroutines)

SIMULINK: Used to Model, Analyze and Simulate Dynamic Systems using block diagrams

Problem :

Customers arrive at a milk booth for the required service. Assume that inter – arrival and service time are constants and given by 1.5 and 4 minutes respectively. Simulate the system by hand computations for 14 minutes.

(i) What is the waiting time per customer?

(ii) What is the percentage idle time for the facility?

(Assume that the system starts at t = 0)

Solution :

First customer arrives at the service center at t = 0
His departure time after getting service = 0 + 4 = 4 minutes.
Second customer arrives at time t = 1.5 minutes
he has to wait = 4 – 1.5 = 2.5 minutes.
Third customer arrives at time t = 3 minutes
he has to wait for = 8-3 = 5 minutes
Fourth customer arrives at time t = 4.5 minutes and he has to wait for 12 – 4.5 = 7.5 minutes.
During this 4.5 minutes, the first customer leaves in 4 minutes after getting service and the second customer is getting service.
Fifth customer arrives at t = 6 minutes
he has to wait 14 – 6 = 8 minutes
Sixth customer arrives at t = 7.5 minutes
he has to wait 14 – 7.5 = 6.5 minutes
Seventh customer arrives at t = 9 minutes
he has to wait 14 – 9 = 5 minutes
During this 9 minutes the second customer leaves the service in 8th minute and third person is to get service in 9th minute.
Eighth customer arrives at t = 10.5 minutes
he has to wait 14 – 10.5 = 3.5 minutes
Nineth customer arrives at t = 12 minutes
he has to wait 14 – 12 = 2 minutes
But by 12th minute the third customer leaves the Service
10th Customer arrives at t = 13.5 minutes
□he has to wait 14-13.5 = 0.5 minute
From this simulation table it is clear that

(i) Average waiting time for 10 customers = $\frac{2.5+5+7.5+8+6.5+5.0+3.5+2+0.5}{10}$

$= \frac{40.5}{10} = 4.05$

(ii) Average waiting time for 9 customers who are in waiting for service $\frac{40.5}{9}$ = 4.5 minutes.

But the average service time is 4 minutes which is less that the average waiting time, the percentage of idle time for service = 0%

Tutorial Questions

1. Solve the following sequence problem given optimal solution when passing is not allowed

Jobs					
Operator	1	2	3	4	5
1	6	2	5	2	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

2. Six jobs are to be processed on three machines A, B, C with the order of processing jobs as BCA

Job	U	V	W	X	Y	Z
Proc,time on machine A	12	10	9	14	7	9
Proc,time on machine B	7	6	6	5	4	4
Proc,time on machine C	6	5	6	4	2	4

The suggested sequence is Y-W-Z-V-U-X. Find out the elapsed time for the sequence suggested. Is it optimal? If it is not optimal, then find out the optimal sequence and the minimum total elapsed time associated with it.

3. A book binder has one printing press, one binding machine and manuscripts of 7 different books The time required for performing printing and binding operations for different books are shown below

Book	1	2	3	4	5	6	7
Printing time (hr)	20	90	80	20	120	15	65
Binding time(hrs)	25	60	75	30	90	35	50

Decide the optimum sequence of processing of books binder to minimize the total time required to bring out all the books

Assignment Questions

1. A bakery keeps stock of a popular brand of cake. Previous experience show the daily demand pattern for the item with associated probabilities as given,Use the following sequence of random numbers to simulate the demand for next 10 days Random numbers: 25,39,65,76,12,05,73,89,19,49 Also estimate the daily average demand for the cakes on the basis of the

Daily damand (number)	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

2. Define simulation why simulation uses. Give one application area when thistechnique is used in practice

Previous Question Papers

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
Department Of Mechanical Engineering
B.Tech IVth Year 1 Semester
Operations Research
MODEL PAPER 1

PART A (25 Marks)

1. a. Write the scope of Operations research (2M)
   b. Write the Applications of Operations Research (3M)
   c. Write the different Types in Transportation problem (2M)
   d. Define the following i)Alternative optimum solution
   ii)unbounded solution
   iii)Slack variable (3M)
   e. Discuss the practical application of assignment problem (2M)
   f. Discuss the steps of Hungarian method (3M)
   g. What is dynamic programming (2M)
   h. What is Kendall Notation .Give the classification of queuing system based on Kendall Notation (3M)
   i. Define inventory (2M)
   j. Find the value of the game (3M)

6	9
8	4

PART B (50 Marks)

2. a) Let us consider a company making single product. The estimated demand for the product for the next four months are 1000,800,1200,900 respectively. The company has a regular time capacity of 800 per month and an overtime capacity of 200 per month. The cost of regular time production is Rs.20 per unit and the cost of overtime production is Rs.25 per unit. The company can carry inventory to the next month and the holding cost is Rs.3/unit/month the demand has to be met every month. Formulate a linear programming problem for the above situation.

b) What are applications of OR

OR

3. Solve the following LPP by Big-M penalty method
   Minimize $Z = 5x_1 + 3x_2$
   S.T $2x_1 \quad 4x_2 \;\delta\; 12, 2x_1 + 2x_2 = 10, 5x_1 + 2x_2 \;\varepsilon\; 10$
   and $x_1, x_2 \;\varepsilon\; 0$

4. A company has factories at F_1, F_2 and F_3 that supply products to ware houses at W_1, W_2 and W_3

.The weekly capacities of the factories are 200,160 and 90 units. The weekly warehouse requirements are 180,120 and 150/units respectively. The unit shipping costs in rupees are as follows find the optimal solution

	W1	W2	W3	supply
F1	16	20	12	200
F2	14	8	18	160
F3	26	24	16	90
Demand	180	120	150	

OR

5. Different machines can do any of the five required jobs with different profits ring from each assignment as shown in adjusting table. Find out maximum profit possible through optimal assignment

.

Jobs	Machines				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

6. Solve the following sequence problem given optimal solution when passing is not allowed

Machines	Jobs				
	A	B	C	D	E
M1	11	13	9	16	17
M2	4	3	5	2	6
M3	6	7	5	8	4
M4	15	8	13	9	11

OR

7. Machine A costs of Rs:80,000. Annually operating cost are Rs:2,000 for the first years and they increase by Rs:15,000 every years (for example in the fourth year the operating cost are Rs:47,000) .Determine the least age at which to replace the machine. If the optional replacement policy is followed (a)What will be the average yearly cost of operating and owing the machine (Assume that the reset value of the machine is zero when replaced, and that future costs are not discounted

b)Another machine B cost Rs:1,00,000.Annual operating cost for the first year is Rs:4,000 and they increase by Rs:7,000 every year .The following firm has a ma(chine of type A which is one year old. Should the firm replace it with B and if so when?

(c)Suppose the firm is just ready to replace the M/c A with another M/c of the same type, just the the firm gets an information that the M/c B will become available in a year .What should firm do?

8. Obtain the optimal strategies for both pensions and the value of the game for two persons zero sum game whose payoff matrix is as follows.

	player-B		
		B1	B2
Player-A	A1	1	-3
	A2	3	5
	A3	-1	6
	A4	4	1
	A5	2	2
	A6	-5	0

OR

9. The production department of a companyrequired 3,600kg If raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs.36 and the cost of carrying inventory is 25% for the investment in the inventories, the price is Rs.10/kg. help the purchase manager to determine and ordering policy for raw material, determine optimal lot size

10. Customers arrive at box office windows being manned by a single individual according to a poisson input process with a mean rate of 20/hr. the time required to same a customer has an exponential distribution with a mean of 90 sec. Find the average waiting time of customers. Also determine the average number of customers in the system and average queue length

OR

11. a)What is simulation? Discuss application of simulation? b)Minimize $z=y_1^2+y_2^2+y_3^2$

subjected to $y_1+y_2+y_3=10$ and $y1, y_2, y_3 \geq 0$ solve using Bellman's principle

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
Department Of Mechanical Engineering
B.Tech 4th Year 1 Sem
Operations Research
MODEL PAPER 2

PART A **(25 Marks)**

1. a. Define Operations research (2M)
   b. Write the industrial Applications of Operations Research (3M)
   c. Write the different Types in Transportation problem (2M)
   d. Write algorithm for Northwest corner method (3M)
   e. Discuss the practical application of assignment problem (2M)
   f. Discuss the steps of Hungarian method (3M)
   g. What is difference between balanced and unbalanced problems in the Assignment problems (2M)
   h. What is Kendall Notation .Give the classification of queuing system based on Kendall Notation (3M)
   i. Define inventory (2M)
   j. Find the value of the game (3M)

	6	2	4	
	2	3	3	
	5	2	6	

PART B **(50 Marks)**

2.a) Solve the following LP problem using graphical method

Maximize Z == $-x_1+2x_2$

Subjected to$x_1-x_2 \leq -1$

$-0.5x1-x2 \leq 2$ $x_1,x_2 \geq 0$

b) Explain the advantages of OR

OR

3 a.) Explain what is meant by degeneracy in LPP? How can this be solved?

b.) Solve the following LP problem by two phase method. Maximize Z = $5x_1+3x_2$

subjected to $3x_1+2x_2 \geq 3$

$x_1+4x_2 \geq 4$

$x_1+x_2 \leq 5$

$x_1+x_2 \geq 0$

4. a) Solve the following assignment problem to minimize the total time of the

Operator

Jobs					
Operator	1	2	3	4	5
1	6	2	5	2	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

b) Write the Mathematical representation of an assignment model?

OR

5. a). Briefly explain about the assignment problems in OR and applications of assignment in OR?

b) What do you understand by degeneracy in a transportation problem?

6. A book binder has one printing press, one binding machine and manuscripts of 7 different books The time required for performing printing and binding operations for different books are shown below

Book	1	2	3	4	5	6	7
Printing time (hr)	20	90	80	20	120	15	65
Binding time(hrs)	25	60	75	30	90	35	50

Decide the optimum sequence of processing of books binder to minimize the total time required to bring out all the books

OR

7. Six jobs are to be processed on three machines A, B, C with the order of processing jobs as BCA

Job	U	V	W	X	Y	Z
Proc,time on machine A	12	10	9	14	7	9
Proc,time on machine B	7	6	6	5	4	4
Proc,time on machine C	6	5	6	4	2	4

The suggested sequence is Y-W-Z-V-U-X. Find out the elapsed time for the sequence suggested. Is it optimal? If it is not optimal, then find out the optimal sequence and the minimum total elapsed time associated with it.

8. Define group replacement policy.

b) A computer contains 10000 resistors. When any resistor fails, it is replaced the cost of replacing a resistor individually is Rs.1 only. If all the resistors are replaced at the same time, cost per resistor would be reduced to 35 paisa. The % of surviving resistors say S(t) at the end of month t and the P(t) the probability of failure during

The month t is.

t	0	1	2	3	4	5	6
S(t)	100	97	90	70	30	15	0
P(t)	-	0.03	0.07	0.2	0.4	0.15	0.15

What is the optimal replacement policy?

OR

9. a)Explain the terms

i) Maxmin criteria and Minimax criteria ii) Strategies: Pure and mixed strategies.

b) Solve the following game graphically

	Player B		
Player A	B_1	B_2	B_3
A_1	1	3	11
A_2	8	5	2

10.Customers arrive at box office windows being manned by a single individual according to a poisson input process with a mean rate of 20/hr. the time required to save a customer has an exponential distribution with a mean of 90 sec. Find the average waiting time of the customers. Also determine the average number of customers in the system and average queue length

OR

11 a) State and explain the Bellman"s principle of optimality.

b) Solve the LPP by dynamic programming approach Maximize z = 4x1 + 14x2

such that 2x 1 + 7x2 ≤ 21
7x1 + 2x2 ≤ 21 x1, x2 ≥ 0

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

Department Of Mechanical Engineering

B.Tech 4th Year 1 Sem

Operations Research

MODEL PAPER 3

PART A **(25 MARKS)**

1. a. Define Operations research (2M)
   b. Write the assumptions in Linear programming (3M)
   c. Write the formula for EOQ for the purchase model without shortages (2M)
   d. Write algorithm for Least Cost Cell method (3M)
   e. Discuss the practical application of assignment problem (2M)
   f. Discuss the steps of Hungarian method (3M)
   g.What is dominance property (2M)
   h.Distinguish between breakdown maintenance and preventive maintenance (3M)
   i. Define dynamic programming (2M)
   j. Find the value of the game (3M)

6	**2**	**4**
2	**3**	**3**
5	**2**	**6**

PART B **(50 Marks)**

2. Solve the following LPP problem by Two phase method Max $Z=2x_1+3x_2+5x_3$

   S.T $3x_1+10x_2+5x_3\leq15$

   $33x_1-10x_2+9x_3\leq33$

   $x_1+2x_2+3x_3\geq4$

   $x_1, x_2, x_3\geq0$

OR

3.a) Define the LPP. Give an example

b) Solve the following LPP using graphical method and verify by Simplex method

Maximize $Z=10x_1+8x_2$

S.T $x_1+2x_2\leq1000$

$x_1\leq300$

$x_2\leq500$ and $x_1, x_2, \geq0$

4. a) Give the mathematical formulation of Transportation problem

b) Use Vogel's approximate method to obtain an initial basic feasible solution of a transportation problem and find the optimal solution

Warehouse / Factory	W	X	Y	Z	Supply
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

OR

5. Six jobs go first on machine A, then on machine B and last on a machine C. The order of completion of jobs have no significance. The following table gives machine time for the six jobs and the three machines. Find the sequence of jobs that minimizes elapsed time to complete the jobs.

Jobs	Processing Time		
	Machine A	Machine B	Machine C
1	8	3	8
2	3	4	7
3	7	5	6
4	2	2	9
5	5	1	10
6	1	6	9

6. The data collected in running a Machine the cost of which is Rs: 60,000 are

Resale value	1	2	3	4	5
Resale value (R	42,000	30,000	20,400	14,400	9,650
Cost of Spares (	4,000	4,270	4,880	5,700	6,800
Cost of Labor	14,000	16,000	18,000	21,000	25,00

Find the time when the machine should be replaced?

OR

7. Find the most economic batch quantity of a product on machine if the production rate of the item on the machine is 300 pieces per day and the demand is uniform at the rate of 150 pieces/day. The set up Cost is Rs.300 per batch and the cost of holding one item in inventory is Rs.0.81/per day. How will the batch quantity vary if the machine production n rate was infinite?

8. a)Explain the terms i)Rectangular games ii) type of Strategies

b) Solve the following game graphically where pay off matrix for player A has been prepared

1	5	-7	4	2
2	4	9	-3	1

OR

9. A dealer supplies you the following information with regards to a product that he deals in annual demand =10,000 units, ordering cost Rs.10/order. Price Rs.20/unit. Inventory carrying cost is 20% of the value of inventory per year. The dealer is considering the possibility of allowing some back orders to occur. He has estimated that the annual cost of back ordering will be 25% of the value of inventory

   a. What should be the optimum no of units he should buy in 1 lot?
   b. What qty of the product should be allowed to be back ordered
   c. What would be the max qty of inventory at any time of year
   d. Would you recommend to allow backordering? If so what would be the annual cost saving by adopting the policy of back ordering.

10. a) Explain how the queues are classified and give their notations
    b) In a bank, cheques are cashed at a single "teller" counter. Customers arrive at the counter in a Poisson manner at an average rate of 30 customers/hr. The teller takes on an average 1.5 minutes to cash a cheque. The service time has been shown to be exponentially distributed.
    i) Calculate the percentage of time the teller is busy
    ii) Calculate the average time a customer is expected to wait.

OR

11. Use dynamic programming to solve the following
    LPP Max $z= 3x_1+5x_2$

    Subjected to $x_1 \leq 4.$

    $X_2 \leq 6,$

    $3x_1+2x_2 \leq 18$,

    $x1,x2 \geq 0$

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
Department Of Mechanical Engineering
B.Tech 4th Year 1 Sem
Operations Research

MODEL PAPER 4

PART A **(25 Marks)**

1. a. Define Operations research (2M)
   b. What is simulation and what is the need of simulation (3M)
   c. What is surplus variable (2M)
   d. Write algorithm for Northwest corner method (3M)
   e. Discuss the practical application of Transportation problem (2M)
   f. Discuss the steps of Hungarian method (3M)
   g. What is difference between balanced and unbalanced problems in the Assignment problems (2M)
   h. Define the following i)balking
   ii) Reneging
   iii) dynamic programming (3M)
   i. Define strategy (2M)
   j. Find the value of the game (3M)

1	-1	3	-1	5
-2	2	-2	4	-2

PART B **(50 Marks)**

2. a)Write the applications and scope of OR

   b) Use Big-M method solve the following

   Max $Z=6x_1+4x_2$

   Subjected to $2x_1+3x_2\leq30$, $3x_1+2x_2\leq24$, $x_1+x_2\geq3$, $x_1, x_2 \geq0$

OR

3. Solve the following LPP by Two phase method Max $z =2x_1+3x_2+5x_3$

   subjected to $3x_1+10x_2+5x_3\leq15$

   $33x_1-10x_2+9x_3\leq33$

   $x_1+2x_2+3x_3\geq4$

   $x_1,x_2, x_3 \geq0$

4.a) What do you understand by degeneracy in a transportation problem?

b) Obtain initial solution in the following transportation problem by using VAM and LCM

Source	D1	D2	D3	D4	D5	Availability
S1	5	3	8	6	6	1100
S2	4	5	7	6	7	900
S3	8	4	4	6	6	700
Requirement	800	400	500	400	600	

OR

5. Different machines can do any of the five required jobs with different profits resulting from each assignment as shown in the adjusting table. Find out maximum profit possible through optimal assignment.

Jobs	Machines				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

6. A salesman has to visit five cities A,B,C,D,E. The intercity distances are tabulated below

	A	B	C	D	E
A	-	12	24	25	15
B	6	-	16	18	7
C	10	11	-	18	12
D	14	17	22	-	16
E	12	13	23	25	-

Find the shortest route covering all the cities.

OR

7.a) Explain the terminology of sequencing techniques in operations research?

b) Solve the following sequence problem, given an optimal solution when passing is not allowed

Machines	Jobs				
	A	B	C	D	E
M1	11	13	9	16	17
M2	4	3	5	2	6
M3	6	7	5	8	4
M4	15	8	13	9	11

8.a) Purchase manager places order each time for a lot of 500 no of particular item from the available data the following results are obtained, inventory carrying 40%, ordering cost order Rs.600, cost per unit Rs.50 annual demand 1000 find out the loser to the organization due to his policy

b) What are inventory models? Enumerate various types of inventory models and describe them briefly

OR

9. a)What are characteristics of a game?

b) Reduce the following Game by dominance and the fid the game value

PlayerA					
		I	II	III	IV
	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

10. A bakery keeps stock of a popular brand of cake. Previous experience show the daily demand pattern for the item with associated probabilities as given

Daily damand (number)	0	10	20	30	40	50
Probability	0.01	0.20	0.15	0.50	0.12	0.02

use the following sequence of random numbers to simulate the demand for next 10 days Random numbers: 25,39,65,76,12,05,73,89,19,49 Also estimate the daily average demand for the cakes on the basis of the

OR

11. Solve using dynamic programming

Max $z= 50x_1+100x_2$

S.T $2x_1+3x_2\leq 48$

$x_1+3x_2\leq 42$

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
Department Of Mechanical Engineering
B.Tech 4th Year 1 Sem
Operations Research

MODEL PAPER 5

PART A **(25 Marks)**

1. a. Define scope of Operations research (2M)
   b. Write the advantages of simulation (3M)
   c. What is artificial variable (2M)
   d. Write algorithm for LCM method (3M)
   e. Discuss the practical application of Transportation problem (2M)
   f. Write any three applications of Bellman's principle of optimality (3M)
   g. What is difference between balanced and unbalanced problems in the Assignment problems (2M)
   h. Write the applications of Travelling salesman problem (3M)
   i. Define pure strategy (2M)
   j. Discuss the steps of Hungarian method (3M)

PART B **(50 Marks)**

2. a) A firm produces three types of biscuits A,B,C it packs them in arrangement of two sizes 1 and 11. The size 1 contains 20 biscuits of type A, 50 of type B and 10 of type C. the size 11 contains 10 biscuits of type A, 80 of type B and 60 of type C. A buyer intends to buy at least 120 biscuits of type A, 740 of type B and 240 of type C. Determine the least number of packets he should buy. Write the dual LP problem and interrupt your answer

   b) Solve the following LPP using graphical method and verify by Simplex method

   Maximize $Z=10x_1+8x_2$

   S.T $x_1+2x_2\leq1000$

   $x_1\leq300$

   $x_2\leq500$ and $x_1, x_2, \geq0$

OR

3. a) Explain what is meant by degeneracy in LPP? How can this be solved? b)Solve the following LP problem by graphically

   Maximize $Z=2x_1+x_2$

   S.T $x_1+2x_2\leq10$, $x_1+x_2\leq 6, x_1-x_2\leq 2, x_1-2x_2\leq 1$ $x_1,x_2\geq 0$

4. a) State the assignment problem mathematically.

   b) For the assignment table, find the assignment of salesmen to districts that will result in maximum sales

Sales people \ Districts	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

OR

5. a) What do you understand by degeneracy in a transportation problem?

b)A company has three plants at locations A.B.C which supply to Warehouse located at D,E,F,G and H. Monthly plant capacities are 800,500, and 900 respectively. Monthly warehouse requirements are 400,500,400 and 800units.Unit Transportation cost in rupees is

	D	E	F	G	H
A	5	8	6	6	3
B	4	7	7	6	5
C	8	4	6	6	4

Determine the optimum distribution for the company in order to minimize total transportation cost by NWCR

6. a) State Group of replacement policy

b) The following failure rates have been observed for a certain type of light bulbs

End of week	Probability of failure date
1	0.05
2	0.13
3	0.25
4	0.43
5	0.68
6	0.88
7	0.96
8	1.00

The cost of replacing an individual failed bulb is Rs.1.25.the decision is made to replace all bulbs simultaneously at fixed intervals and also to replace individual bulbs as they fall in service. If the cost of group replacement is 30 paise per bulb, what is the best interval between group replacements? At what group replacement price per bulb would a policy of strictly individual replacement become preferable to the adopted policy?

OR

7. a) A firm is considering the replacement of a machine, whose cost price is Rs.12,200 and its shop value is Rs.200. From experience the running (maintenance and operating) costs are found to be as follows.

Year	1	2	3	4	5	6	7	8
Ruuning cost	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced?

b) Explain two person zero sum game and n person game?

8. The demand for a purchased item 1000 units per month and shortages are allowed. If the unit cost is Rs. 1.50 per unit, the cost of making one purchase is Rs.600,the holding cost for one unit is Rs.2 per year and one shortage isRs.10 per year. Determine

i) The optimum purchase quantity

ii) The number of orders per year

iii) The optimal total yearly cost

OR

9. a)Obtain the optimal strategies for both pensions and the value of the game for two persons zero sum game whose payoff matrix is as follows.

		player-B	
		B1	B2
Player-A	A1	1	-3
	A2	3	5
	A3	-1	6
	A4	4	1
	A5	2	2
	A6	-5	0

b) Explain pay of matrix and types of strategy in game theory?

10. a) Define simulation why simulation uses. Give one application area when this technique is used in practice

b) Use dynamic programming to solve the following LPP Max $z= 3x_1+5x_2$

Subjected to $x_1 \leq 4.$

$X_2 \leq 6,$

$3x_1+2x_2 \leq 18$,

$x1,x2 \geq 0$

OR

11. a)What are the applications of the dynamic programming?.Explain Bellman's principle of optimality.

b)Using dynamic programming approach solve the below problem

Maximize $z = 8x_1 + 7x_2$

S.T $2x_1 + x_2 \leq 8, 5x_2 + 2x_2 \leq 15, x_1, x_2 \geq 0$

R15

Code No: R15A0330

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

IV B.Tech I Semester Supplementary Examinations, October 2020

Operations Research

(ME)

Roll No										

Time: 2 hours **Max. Marks: 75**

Answer Any **Four** Questions

All Questions carries equal marks

1 Solve the following LPP by simplex method

Minimize $Z=8x_1 - 2x_2$.

Subject to $-4x_1+ 2x_2 \leq 1$

$5x_1 - 4x_2 \leq 3$

$x_1, x_2 \geq 0$.

2 Max $Z= 2 X_1+ 5X_2$

Subject to: $2X_1+ X_2 \leq 43$

$3X_1+2X_2 \leq 46$

$X_1, X_2 \geq 0$

Use Graphical method approach to solve LPP.

3 Solve the following transportation problem:

		To Destination				Availability
		1	2	3	4	
From Origins	1	15	0	20	10	50
	2	12	8	11	20	50
	3	0	16	14	18	100
Requirement		30	40	60	70	200

4 Find the sequence that minimizes the total elapsed time required to complete the following tasks on machines M_1, $M_{2,}$ M3 in the order M_1M_2 M3. Also, find the minimum total elapsed time.

Task	1	2	3	4	5	6	7	8	9
M_1	4	9	6	10	6	12	8	3	8
M2	6	4	8	9	4	6	2	6	4
M3	10	12	9	11	14	15	10	14	12

5 A field owner finds from this past records the cost of a running a truck those purchase price is RS.6000 of a given below. The maintenance cost and resale value per year of a machine below:

Year	1	2	3	4	5	6	7
Running lost.	1000	1200	1400	1800	2300	2800	3400
Resale value or soap value	3000	1500	750	375	200	200	200

What are the optimal replacement period?

6 Using Dominance property solve the game

$$\begin{bmatrix} I & II & III & IV \\ -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{bmatrix}$$

7 What are the various costs involved with the inventory? Explain.

8 Define simulation. Why is simulation used? Give one application area where this technique is used practice?

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R17

Code No: R17A0333

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

IV B.Tech I Semester Regular Examinations, February 2021

Operations Research

(ME)

Roll No										

Time: 2 hours 30 min **Max. Marks: 70**

Answer Any **Five** Questions

All Questions carries equal marks.

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1 Describe various operations research models elaborately? **[14M]**

2 Use Simplex Method To minimize $Z = x_1 - 2x_2 + 3x_3$ **[14M]**

Subject to the constraints

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1, x_2, x_3 \geq 0$$

3 Solve the following transportation problem, in which cell entries represent unit costs **[14M]**

		To			
		A	B	C	Available
From	I	2	7	4	5
	II	3	3	1	8
	III	5	4	7	7
	IV	1	6	2	14
	Reqd.	7	9	18	34

4 Solve the following transportation problem for optimum transportation cost. **[14M]**

	Destination				**Available**
	A	**B**	**C**	**D**	
1	19	30	50	10	7
2	70	30	40	60	9
3	40	8	70	20	18
Demand	5	8	7	14	

5 Consider (two persons, zero sum) game matrix which represents pay off to the player 'A'. Find the optimal strategy, if any. **[14M]**

		Player B		
		I	II	III
Player A	I	-3	-2	6
	II	2	0	2
	III	5	-2	-4

6 Find game value of the following payoff matrix. **[14M]**

	Player B			
Player A	18	4	6	4
	6	2	13	7
	11	5	17	3
	7	6	12	2

7 A stock list has to supply 400 units of a product every Monday to his customers. He gets the product at Rs. 50 per unit from the manufacturer. The cost of ordering and transportation from the manufacturer is Rs.75 per order. The cost of carrying inventory is 7.5% per year of the product. Find **[14M]**

i) The economic lot size

ii) The total optimal cost(including the capital cost) .

8(a) What is simulation? Explain the phases of simulation? **[7M]**

(b) What are the features of simulation languages? Explain? **[7M]**

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R15

Code No: R15A0330

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)
IV B. Tech I Semester Regular Examinations, November 2018
Operations Research
(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

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PART-A (25 Marks)

1). a What are the different models in operations research? **[2M]**
b What is meant by unique, infinite, unbounded, infeasible solution? **[3M]**
c What is meant by degeneracy in transportation? **[2M]**
d Define sequencing and state what are the assumptions **[3M]**
e What are the two types of replacements? **[2M]**
f Define the principle of Max min – Min max? **[3M]**
g Define simulation **[2M]**
h Define a queue. What is the customer behaviour? **[3M]**
i State applications of simulation. **[2M]**
j What is dynamic programming? **[3M]**

PART-B (50 MARKS)

SECTION-I

2 Use graphical method to solve the following problem: **[10M]**

$Z_{Max} = 4x_1 + 10x_2$.

Subject to $2x_1 + x_2 \leq 50$
$2x_1 + 5x_2 \leq 100$
$2x_1 + 3x_2 \leq 90$.
$x_1, x_2 \geq 0$

OR

3 Solve the following LPP **[10M]**

Maximize $Z = 15x_1 + 6x_2 + 9x_3 + 2x_4$
s.t $2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$,
$3x_1 + x_2 + 3x_3 + 25x_4 \leq 20$,
$7x_1 + x_4 \leq 70$,
x_1, x_2, x_3 and $x_4 \geq 0$

SECTION-II

4 Solve the following assignment problem. **[10M]**

1	3	2	8	8
2	4	3	1	5
5	6	3	4	6
3	1	4	2	2
1	5	6	5	4

OR

5 a) What are the assumptions made in sequencing problems? **[10M]**

b) Find the sequence that minimizes the total elapsed time required to complete the following tasks:

Tasks		A	B	C	D	E	F	G
Processing	Machine 1	3	8	7	4	9	8	7
	Machine 2	4	3	2	5	1	4	3
	Machine 3	6	7	5	11	5	6	2

SECTION-III

6 A field owner finds from this past records the cost of a running a truck those purchase price is Rs.6000 of a given below. The maintenance cost and resale value per year of a machine whose purchase price is Rs.7000 is given below: **[10M]**

Year	1	2	3	4	5	6	7
Running lost.	1000	1200	1400	1800	2300	2800	3400
Resale value or soap value	3000	1500	750	375	200	200	200

OR

7 Solve the following game: **[10M]**

	I	II	III	IV
I	6	8	3	13
II	4	1	5	3
III	8	10	4	12
IV	3	6	7	12

SECTION-IV

8 Find the optimal quantity for a product for which price breaks are as follows: **[10M]**

Quantity	Unit cost
$0 \leq Q1 < 500$	Rs. 10
$500 \leq Q2 \leq 750$	Rs. 9.25
$750 \leq Q3$	Rs8.75

The monthly demand for a product is 200 units, the cost of storage is 2 % of the unit cost and the cost of ordering is Rs.350

OR

9 People arrive at a Theatre ticket booth in Poisson distributed arrival rate of 25 per hour. Service time is Constant at 2 minutes. Calculate **[10M]**

a) The mean number in the waiting line.
b) The mean waiting time.
c) The utilization factor.

SECTION-V

10 State the Bellman's principle of optimality in dynamic programming and give a mathematical formulation of a dynamic programming problem? **[10M]**

OR

11 Define simulation. Why is simulation used? Give one application area where this technique is used practice? **[10M]**

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R15

Code No: R15A0330

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

IV B. Tech I Semester Supplementary Examinations, May 2019

Operations Research

(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

***

PART-A (25 Marks)

1). a What is operation research? **[2M]**
b Define Slack, Surplus & Artificial variables. **[3M]**
c What is the difference between Transportation &assignment? **[2M]**
d What are the various types of sequencing problems? **[3M]**
e Define discount factor? **[2M]**
f Define saddle point & value of the game. **[3M]**
g Define the term EOQ **[2M]**
h What are the classifications of Queering Models? **[3M]**
i Define stage. **[2M]**
j State applications of dynamic programming **[3M]**

PART-B (50 MARKS)

SECTION-I

2 A pare mill produces 2 grades of paper namely x and y. Because of raw material restrictions, it cannot produce more than 400 tons of grade x and 300 tons of grade y in a week there are 160 production hours in a week. Of require 0.2 and 0.4 hours to produce a tone of product z , y respectively, with corresponding pretties of Rs. 200 and Rs. 500 per ton .Formulas the above as a LPP it maximize the profit & find the optimum product mix using graphical method. **[10M]**

OR

3 Solve the following LPP by simplex method **[10M]**
Minimize $Z=8x_1 – 2x_2$.
s.t $-4x_1+ 2x_2 \leq 1$
$5x_1 – 4x_2 \leq 3$
$x_1, x_2 \geq 0$

SECTION-II

4 Find the optimal sequence for processing jobs through the two Machines A, B in the order of ABC. Processing times are given below. **[10M]**

JOBS	1	2	3	4	5	6	7	8
A	14	19	16	20	16	22	18	13
B	16	14	18	19	14	16	12	16

Find the optimal sequence and total elapsed times.

OR

5 Solve the following transportation problem: **[10M]**

		To Destination				Availability
		1	2	3	4	
From Origins	1	15	0	20	10	50
	2	12	8	11	20	50
	3	0	16	14	18	100
Requirement		30	40	60	70	200

SECTION-III

6 The maintenance cost and resale value per year of a machine whose purchase price is Rs.7000 is given below: **[10M]**

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs.	900	1200	1600	2100	2800	3700	4700	5900
Resale value in Rs.	4000	2000	1200	600	500	400	400	400

When should the machine be replaced?

OR

7 Solve the following game using Dominance property **[10M]**

$$\begin{array}{cccc} & I & II & III \\ A_1 & 1 & 7 & 2 \\ A_2 & 6 & 2 & 7 \\ A_3 & 6 & 1 & 6 \end{array}$$

SECTION-IV

8 Find the optimal quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering cost per order is Rs. 180.00. The unit costs are given below. **[10M]**

Quantity	Unit cost
$0 \leq Q_1 < 500$	Rs. 25
$500 \leq Q_2 < 1500$	Rs. 24.80
$1500 < Q_3 < 3000$	Rs. 24.60
$3000 < Q_4$	Rs. 24.40

OR

9 Job arrival at a work station in a manufacturing plant is in Poisson fashion at an average of 5 per hour. The time to machine one job is an exponential distribution with a mean time of 20 minutes. What is the expected time a job has to wait at the workstation? What is the probability that there will be more than four jobs? **[10M]**

SECTION-V

10 State the Bellman's principle of optimality in dynamic programming and give a mathematical formulation of a dynamic programming problem? **[10M]**

OR

11 What are advantages and disadvantages of simulation? **[10M]**

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Code No: **R15A0330**

R15

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

IV B.Tech I Semester Supplementary Examinations, February 2021

Operations Research

(ME)

Roll No										

Time: 2 hours 30 min **Max. Marks: 75**

Answer Any **Five** Questions

All Questions carries equal marks.

***

(1) Discuss the following terms in brief **(15M)**

(a) Basic Solution (b) Unbounded Solution (c) Degenerate Basic feasible solution

(d) Extreme points (e) optimal solution

(2) Solve the following Linear Programming Problem using Simplex Method **(15M)**

Minimize $Z = X_1 - 3X_2 + 2X_3$

Subjected to the constraints: $3X_1 - 2X_2 + 3X_3 \leq 7$

$-2X_1 + 2X_2 \leq 12$

$-4X_1 + 3X_2 + 8X_3 \leq 15$

$X_1, X_2, X_3 \geq 0$

(3) Consider the following table and solve the Problem for the minimization of cost of Assignment. **(15M)**

Job → / Machine ↓	1	2	3	4
A	18	24	28	32
B	8	13	17	19
C	15	15	19	22

(4) A Machine operator has to perform two operations (Turning and Threading) on Lathe Machine for different jobs. The time required to perform the operations on each job are shown in table below. Deter mine the sequence in which the jobs are to be processed in order, so that the time required to complete all the jobs is minimized. **(15M)**

Job	**1**	**2**	**3**	**4**	**5**	**6**
Time taken for Turning (Hrs) (Machine-A)	4	13	6	3	15	12
Time taken for Threading(Hrs) (Machine-B)	9	11	15	7	4	2

(5) A machine is purchased for Rs. 3000 and running costs are estimated at Rs.800 for each of the first 5 years and thereafter it increases every year by Rs. 200 from sixth to tenth year. If the money is worth 15% per year, Determine the year at which machine should be replaced. **(15M)**

(6) (a) Explain the following terms in brief **(5M)**

(i) Pure strategy & Mixed strategy (ii) Saddle Point

(b) Consider the following game matrix that represents the payoff to the player A. Find the Optimal strategy. **(10M)**

		Player B		
Player A		**I**	**II**	**III**
	I	-3	-2	6
	II	2	0	2
	III	5	-2	-4

(7) A manufacturer has to supply 15,000 units/year. The ordering cost is Rs. 200 and the holding cost is Rs. 3.00 / year. If the replacement is instantaneous and no shortages are allowed. Assume purchase cost as Rs.1/unit. Find

(i) Optimum run size

(ii) Optimum scheduling period

(iii) The no. of orders per year **(15M)**

(8) Explain in detail about Bellman's Principle of optimality and list out the applications of Dynamic programming. **(15M)**

Code No: R15A0330 **R15**

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)

IV B.Tech I Semester Regular/Supplementary Examinations, November 2019

Operations Research

(ME)

Roll No										

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B
Part A is compulsory which carriers 25 marks and Answer all questions.
Part B Consists of 5 SECTIONS (One SECTION for each UNIT). Answer FIVE Questions, Choosing ONE Question from each SECTION and each Question carries 10 marks.

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PART-A (25 Marks)

1). a Define solution to LPP and basic solution **[2M]**
b What is meant by degeneracy in linear programming? **[3M]**
c Define feasible solution in transportation problem. **[2M]**
d Give a mathematical formulation of the transportation problem. **[3M]**
e What is pure strategy? **[2M]**
f Explain Row dominance and column dominance. **[3M]**
g Define the terms: Balking and reneging. **[2M]**
h Explain the concept of EOQ. **[3M]**
i Define the stage and state of a dynamic programming model. **[2M]**
j List out the types of simulation. **[3M]**

PART-B (50 MARKS)

SECTION-I

2 Use the graphical method to solve the following LP problem. **[10M]**

$$\text{Minimize} \quad Z = 3x_1 + 2x_2$$

subject to the constraints:

$$5x_1 + x_2 \geq 10;\ x_1 + x_2 \geq 6;\ x_1 + 4x_2 \geq 12;\ x_1, x_2 \geq 0$$

OR

3 Solve the following LP problem using the simplex method. **[10M]**

$$\text{Maximize} \quad Z = 3X_1 + 2X_2$$

Subject to

$$2X_1 + X_2 \leq 2;\ 3X_1 + 4X_2 \geq 12;\ X_1, X_2 \geq 0$$

SECTION-II

4 A department has five employees with five jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix. **[10M]**

	Employees					
		I	II	III	IV	V
Jobs	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

OR

5 We have six jobs, each of which must go through machines A, B and C in the order ABC. Processing time (in hours) are given in the following table: **[10M]**

Job	:	1	2	3	4	5	6
Machine A	:	8	3	7	2	5	1
Machine B	:	3	4	5	2	1	6
Machine C	:	8	7	6	9	10	9

Determine a sequence for the five jobs that will minimize the elapsed time, idle time on machine A, B and C.

SECTION-III

6 The data on the running costs per year and resale price of equipment A whose purchase price is Rs 2,00,000 are as follows: **[10M]**

Year	1	2	3	4	5	6	7
Running cos (Rs)	30,000	38,000	46,000	58,000	72,000	90,000	1,10,000
Resale value (Rs)	1,00,000	50,000	25,000	12,000	8,000	8,000	8, 000

What is the optimum period of replacement?

OR

7 The maintenance cost and resale value per year of a machine whose purchase price is Rs.7000 is given below: **[10M]**

Year	1	2	3	4	5	6	7	8
Maintenance cost in Rs.	900	1200	1600	2100	2800	3700	4700	5900
Resale value in Rs.	4000	2000	1200	600	500	400	400	400

When should the machine be replaced?

SECTION-IV

8 Find the optimal quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering cost per order is Rs. 180.00. The unit costs are given below. **[10M]**

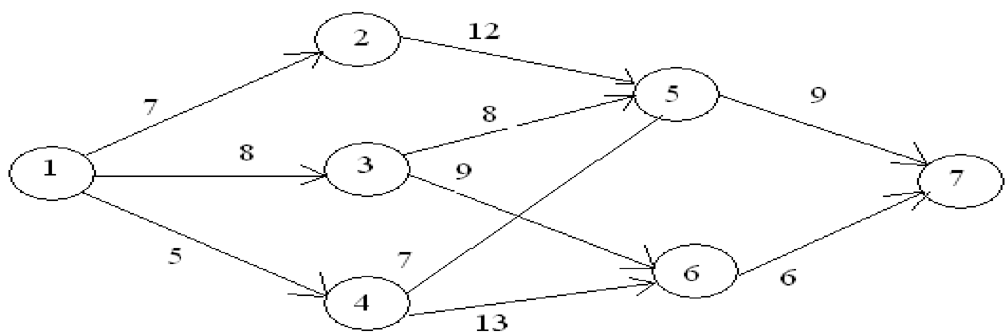
Quantity	Unit cost
$0 \leq Q_1 < 500$	Rs. 25
$500 \leq Q_2 < 1500$	Rs. 24.80
$1500 < Q_3 < 3000$	Rs. 24.60
$3000 < Q_4$	Rs. 24.40

OR

9 On an average 96 patients per 24 hour day require the service of an emergency clinic. Also on average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs.100 per patient treated to obtain an average servicing time of 10 minutes, and that each minute of decrease in this average time would cost Rs.10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from 1 1/3 patient to ½ patient. **[10M]**

SECTION-V

10 Select the shortest highway route between two cities. The network in fig: provides the possible routes between the starting city at node 1 and destination city at node 7. **[10M]**



OR

11 Define simulation. Why is simulation used? Give one application area where this technique is used practice? **[10M]**

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